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Strong convergence theorems for a finite family of asymptotically nonexpansive mappings and semigroups

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Abstract

Strong convergence theorems are obtained for a finite family of asymptotically nonexpansive mappings and semigroups by the modified Mann method.

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1. Introduction

Let K be a nonempty closed convex subset of a Hilbert space H. A mapping $T : K \to K$ is said to be *nonexpansive* if for all $x, y \in K$ we have $\|Tx - Ty\| \leq \|x - y\|$. It is said to be *asymptotically nonexpansive* [2] if there exists a sequence $\{k_n\}$ with $k_n \geq 1$ and $\lim_{n\to\infty} k_n = 1$ such that $\|T^n x - T^n y\| \leq k_n\|x - y\|$ for all integers $n \geq 1$ and all $x, y \in K$. The set of fixed points of T is denoted by $F(T)$.

One parameter family $\mathcal{T} := \{T(t) : t \in \mathbb{R}^+\}$, where $\mathbb{R}^+$ denotes the set of nonnegative real numbers, is said to be *a (continuous) Lipschitzian semigroup on* K [16] *of mappings* from K into K if the following conditions are satisfied:

(1) $T(0)x = x$ for all $x \in K$;
(2) $T(s + t) = T(s)T(t)$ for all $s, t \in \mathbb{R}^+$;
(3) for each $t > 0$, there exists a bounded measurable function $L_t : (0, \infty) \to [0, \infty)$ such that $\|T(t)x - T(t)y\| \leq L_t\|x - y\|, x, y \in K$;
(4) for each $x \in K$, the mapping $T(.)x$ from $\mathbb{R}^+$ into K is continuous.

A Lipschitzian semigroup $\mathcal{T}$ is called *nonexpansive (or contractive)* if $L_t = 1$ for all $t > 0$, and *asymptotically nonexpansive* if $\limsup_{t\to\infty} L_t \leq 1$, respectively. Let $F(\mathcal{T})$ denote the common fixed point set of the semigroup $\mathcal{T}$, i.e., $F(\mathcal{T}) := \{x \in K : T(t)x = x, \forall t > 0\}$. Notice that for an asymptotically nonexpansive semigroup $\mathcal{T}$, we can

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