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A linearly implicit conservative scheme for the coupled nonlinear Schrödinger equation

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Abstract

The coupled nonlinear Schrödinger equation models several intersting physical phenomena. It presents a model equation for optical fiber with linear birefringence. In this paper, we present a linearly implicit conservative method to solve this equation. This method is second order accurate in space and time and conserves the energy exactly. Many numerical experiments have been conducted and have shown that this method is quite accurate and describe the interaction picture clearly.

Keywords: Coupled nonlinear Shrödinger equation; Linearly implicit scheme; Finite difference method; Solitons

1. Introduction

The propagation of pulses with equal mean frequencies in birefringent nonlinear fiber is governed by the coupled nonlinear Schrödinger equation [3]:

$$\begin{aligned} &\mathrm{i}\frac{\partial \Psi_1}{\partial t} + \frac{\partial^2 \Psi_1}{\partial x^2} + (|\Psi_1|^2 + e|\Psi_2|^2)\Psi_1 = 0 \\ &\mathrm{i}\frac{\partial \Psi_2}{\partial t} + \frac{\partial^2 \Psi_2}{\partial x^2} + (e|\Psi_1|^2 + |\Psi_2|^2)\Psi_2 = 0 \end{aligned} \tag{1}$$

where Ψ_1 and Ψ_2 are the wave amplitudes in two polarizations.

Following the discussion in [3], we derive the exact solution of the system in (1) which has the following form:

$$\Psi_j(x,t) = \sqrt{\frac{2\alpha}{1+e}}\,\mathrm{sech}(\sqrt{\alpha}(x-2vt))\,\exp\,\mathrm{i}\{vx - [v^2-\alpha]t\}, \quad j = 1, 2 \tag{2}$$

where α and v are arbitrary constants.

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