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Coupled fixed points of weakly F-contractive mappings in topological spaces

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ABSTRACT

In this work, we introduce a modified notion of weakly F-contractive mappings and prove new coupled fixed point theorems for the mappings which are weakly F-contractive. Also, we give an example to validate the main results in this work.

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1. Introduction

It is well-known that, if X is a compact metric space and $f : X \to X$ is a weakly contractive mapping (see Section 2 for the definition), then f has a fixed point in X (see [1], pp. 17).

In 1969, Furi and Vignoli [2] extended this result to α-condensing mappings in bounded complete metric spaces (see [3] for the definition). A generalized version of the Furi–Vignoli theorem, using the notion of weakly F-contractive mappings in topological spaces, was proved in [4].

On the other hand, in [5], using the KKM mappings, the authors introduced a new concept of lower (upper) semi-continuous functions (see Definition 2.1 in Section 2) which is more general than the classical one. In [6], the authors used this definition of lower semi-continuity to redefine weakly F-contractive mappings (see Definition 2.2 in Section 2) in order to formulate and prove several results on fixed points.

Recently, Bhaskar and Lakshmikantham [7] noted that their coupled fixed point theorems can be used to investigate a large class of problems and have discussed the existence and uniqueness of solutions for a periodic boundary value problem. More recently, Cho et al. [8] studied the solvability of coupled quasi-solutions for the nonlinear operator equations by using the semi-order method.

In this work, we have modified the notions of a weakly F-contractive mapping $f : X \times X \to X$, where X is a topological space (see Definition 2.3 in Section 2). By using the modified definitions of weakly F-contractive mappings, we reformulate and prove a version of the above-mentioned fixed point theorem (Theorem 1 of [6]) for coupled fixed points (see Theorem 3.1). We also prove some coupled fixed point theorems under the assumptions that a certain iteration of a

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