Applied Mathematics Letters 24 (2011) 320–324

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Statistical summability and approximation by de la Vallée-Poussin mean

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ARTICLE INFO

Article history:
Received 22 April 2010
Accepted 3 October 2010

Keywords:
Statistical convergence
Density
λ-density
λ-statistical convergence
Statistical λ-convergence
Approximation theorem

ABSTRACT

In this paper we define a new type of summability method via statistical convergence by using the density and (V, λ)-summability. We further apply our new summability method to prove a Korovkin type approximation theorem.

1. Introduction and preliminaries

The concept of statistical convergence for sequences of real numbers was introduced by Fast [1] and Steinhaus [2] independently in the same year 1951 and since then several generalizations and applications of this notion have been investigated by various authors.

Let $K \subseteq \mathbb{N}$ and $K_n := \{k \leq n : k \in K\}$. Then the *natural density* of K is defined by $\delta(K) = \lim_n n^{-1}|K_n|$ if the limit exists, where $|K_n|$ denotes the cardinality of K_n.

A sequence $x = (x_k)$ of real numbers is said to be *statistically convergent* to ℓ provided that for every $\epsilon > 0$ the set $K_\epsilon := \{k \in \mathbb{N} : |x_k - \ell| \geq \epsilon\}$ has natural density zero, i.e. for each $\epsilon > 0$,

$$\lim_n \frac{1}{n}|\{j \leq n : |x_j - \ell| \geq \epsilon\}| = 0.$$

The idea of λ-statistical convergence was introduced in [3] as follows:

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 0.$$

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x) =: \frac{1}{\lambda_n} \sum_{j \in I_n} x_j$$

where $I_n = [n - \lambda_n + 1, n]$.

A sequence $x = (x_j)$ is said to be (V, λ)-*summable* to a number ℓ (see [4]) if

$$t_n(x) \to \ell \quad \text{as } n \to \infty.$$

In this case ℓ is called the λ-limit of x.

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0893-9659/$ – see front matter
doi:10.1016/j.aml.2010.10.014