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Research Article

Approximation of Solutions for Second-Order m-Point Nonlocal Boundary Value Problems via the Method of Generalized Quasilinearization

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We discuss the existence and uniqueness of the solutions of a second-order m-point nonlocal boundary value problem by applying a generalized quasilinearization technique. A monotone sequence of solutions converging uniformly and quadratically to a unique solution of the problem is presented.

1. Introduction

The monotone iterative technique coupled with the method of upper and lower solutions [1–7] manifests itself as an effective and flexible mechanism that offers theoretical as well as constructive existence results in a closed set, generated by the lower and upper solutions. In general, the convergence of the sequence of approximate solutions given by the monotone iterative technique is at most linear [8, 9]. To obtain a sequence of approximate solutions converging quadratically, we use the method of quasilinearization [10]. This method has been developed for a variety of problems [11–20]. In view of its diverse applications, this approach is quite an elegant and easier for application algorithms.

The subject of multipoint nonlocal boundary conditions, initiated by Bicadze and Samarskiĭ [21], has been addressed by many authors, for instance, [22–32]. The multipoint boundary conditions appear in certain problems of thermodynamics, elasticity and wave propagation, see [23] and the references therein. The multipoint boundary conditions may be understood in the sense that the controllers at the endpoints dissipate or add energy according to censors located at intermediate positions.