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BOUNDARY-VALUE PROBLEMS FOR NONLINEAR THIRD-ORDER q-DIFFERENCE EQUATIONS

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Abstract. This article shows existence results for a boundary-value problem of nonlinear third-order q-difference equations. Our results are based on Leray-Schauder degree theory and some standard fixed point theorems.

1. Introduction

The subject of q-difference equations, initiated in the beginning of the 19th century [1, 6, 19, 22], has evolved into a multidisciplinary subject; see for example [8, 9, 10, 11, 12, 13, 14, 15, 18, 20, 21] and references therein. For some recent work on q-difference equations, we refer the reader to [2, 3, 5, 7, 16, 17, 23]. However, the theory of boundary-value problems for nonlinear q-difference equations is still in the initial stages and many aspects of this theory need to be explored. To the best of our knowledge, the theory of boundary-value problems for third-order nonlinear q-difference equations is yet to be developed.

In this paper, we discuss the existence of solutions for the nonlinear boundary-value problem (BVP) of third-order q-difference equation

$$\begin{gathered} D_q^3 u(t) = f(t, u(t)), \quad 0 \le t \le 1, \\ u(0) = 0, \quad D_q u(0) = 0, \quad u(1) = 0, \end{gathered} \tag{1.1}$$

where f is a given continuous function.

2. Preliminaries

Let us recall some basic concepts of q-calculus [15, 21].

For $0 < q < 1$, we define the q-derivative of a real valued function f as

$$D_q f(t) = \frac{f(t) - f(qt)}{(1-q)t}, \quad D_q f(0) = \lim_{t \to 0} D_q f(t).$$

Higher order q-derivatives are given by

$$D_q^0 f(t) = f(t), \quad D_q^n f(t) = D_q D_q^{n-1} f(t), \quad n \in \mathbb{N}.$$

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