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Research Article

Optimal Approximate Solutions of Fixed Point Equations

S. Sadiq Basha,[1] N. Shahzad,[2] and R. Jeyaraj[3]

[1] *Department of Mathematics, Anna University, Chennai 600 025, India*
[2] *Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*
[3] *Department of Mathematics, St. Joseph's College Higher Secondary School, Trichy 620 002, India*

Correspondence should be addressed to N. Shahzad, nshahzad@kau.edu.sa

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The main objective of this paper is to present some best proximity point theorems for K-cyclic mappings and C-cyclic mappings in the frameworks of metric spaces and uniformly convex Banach spaces, thereby furnishing an optimal approximate solution to the equations of the form $Tx = x$ where T is a non-self mapping.

1. Introduction

Fixed point theorems delve into the existence of a solution to the equations of the form $Tx = x$ where T is a self-mapping. However, when T is a nonself-mapping, the equation $Tx = x$ does not necessarily have a solution, in which case best approximation theorems explore the existence of an approximate solution whereas best proximity point theorems analyze the existence of an approximate solution that is optimal. Indeed, a classical and well-known best approximation theorem, due to Fan [1], contends that if K is a nonempty convex compact subset of a Hausdorff topological vector space E and T is a continuous non-self mapping from K to E, then there exists an element x in K such that $d(x, Tx) = d(A, B)$. Subsequently, many authors, including Prolla [2], Reich [3], and Sehgal and Singh [4, 5], accomplished several appealing extensions and variants of the preceding best approximation theorem. Further, Vetrivel et al. [6] elicited a more generalized result that unifies and subsumes many such results. Despite the fact that best approximation theorems produce an approximate solution to the equation $Tx = x$, they may not render an approximate solution that is optimal. On the contrary, best proximity point theorems are intended to furnish an approximate solution x that is optimal in the sense that the error $d(x, Tx)$ is minimum. Indeed, in light of the fact