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Efficient Chebyshev spectral methods for solving multi-term fractional orders differential equations

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ABSTRACT

In this paper, we state and prove a new formula expressing explicitly the derivatives of shifted Chebyshev polynomials of any degree and for any fractional-order in terms of shifted Chebyshev polynomials themselves. We develop also a direct solution technique for solving the linear multi-order fractional differential equations (FDEs) with constant coefficients using a spectral tau method. The spatial approximation with its fractional-order derivatives (described in the Caputo sense) are based on shifted Chebyshev polynomials $T_{L,n}(x)$ with $x \in (0,L)$, $L > 0$ and n is the polynomial degree. We presented a shifted Chebyshev collocation method with shifted Chebyshev–Gauss points used as collocation nodes for solving nonlinear multi-order fractional initial value problems. Several numerical examples are considered aiming to demonstrate the validity and applicability of the proposed techniques and to compare with the existing results.

1. Introduction

Fractional differential equations (FDEs) appeared in many fields of science and engineering [1–4]. The analytic results on the existence and uniqueness of solutions to the fractional differential equations have been investigated by many authors; among them [4–7]. In general, most fractional differential equations do not have exact analytic solutions, so approximation and numerical techniques must be used.

Finding accurate and efficient methods for solving FDEs has become an active research undertaking. There are several analytic methods [8–11], which have been developed to solve the fractional differential equations. In the last decade or so, extensive research has been carried out on the development of numerical methods which are numerically stable for both linear and nonlinear fractional differential equations. Diethelm et al. [12] presented Predictor corrector method for numerical solution of FDEs. In [13], the authors have proposed an approximate method for numerical solution of a class of FDEs which are expressed in terms of Caputo type fractional derivative. In fact, the method presented in [13] takes advantage of FDEs converting into Volterra-integral equations. Furthermore, the generalization of the Legendre operational matrix to the fractional calculus has been studied in [14]. In [15,16], the spectral tau method for numerical solution of some FDEs is introduced. Recently, Esmaeili and Shamsi [17] introduced a direct solution technique for obtaining the spectral solution of a special family of fractional initial value problems using a pseudo-spectral method, and in [18] Pedas and Tamme developed the spline collocation methods for solving FDEs.

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