

## **Estimation of Ultraviolet-B Radiation Using Fourier Series**

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*Abstract.* The daily estimation of ultraviolet-B (UV-B) radiation intensity is important to provide information to the public about the potential detrimental effects on health from sun exposure. The overall goal of this paper is to initiate the relation between UV-B radiation and global radiation. Because of the periodicity of variations in solar radiation, global and UV-B radiation, their non-dimensional values are expanded in Fourier series. Fourier coefficients were determined by using measured global solar radiation data for four selected Egyptian weather stations. These coefficients were used to calculate UV-B radiation based on global solar radiation for the same stations. A comparison between the observed and calculated UV-B radiation is presented graphically using the error. The comparison shows a good agreement between the observed and estimated UV-B radiation. The method is applied to calculate the UV-B radiation for other stations with no available measurements, using records of global solar radiation at these stations.

### **1. Introduction**

Solar UV radiation, which comprises 8.73% of the solar radiation spectrum (ASTM, 1981), consists of three wavelength band-regions, the UV-A (315-400nm), the UV-B (280-315nm) and the UV-C (100- 280nm), accounting for 5.9%, 1.33% and 1.5% respectively (Iqbal, 1983). Earth's atmosphere significantly modifies the incoming solar rays due to the absorption and scattering action of oxygen, water vapor, carbon dioxide, ozone, water droplets, dust particles and other biospheric constituents of human and volcanic activities. Clear linear correlation between UV-global and solar global is found, particularly in the region of moderate to low solar global values (Koronakis *et al.*, 2002). The

relation between global and UV-B radiation is relatively well known, but there is now an increased need to improve our knowledge of the physics and the climatology of UV-B radiation. The aim of this study is to establish mathematical formulae relating the global solar radiation with UVB-radiation at each of our meteorological stations. The established formulae will be used to estimate the values of UVB- radiation from the observed values of global solar radiation at the same station and also for the neighboring stations.

## 2. Data and Methodology

### A. Station Network

The first measurement location is on the coast of Red Sea at Hurghada, measurements started in January 1994. The second location is on the coast of Mediterranean Sea at El-Arish, 32m above the sea level, measurements started in April 1998. The third location is at Aswan in the south of Egypt, 192m above the sea level, measurements started in August 1998. The fourth location is at South Valley in Upper Egypt, the measurements started in April 2000. The last location at Rafah, about 45 kilometers east of El-Arish station, measurements started in June 2000. Global solar radiation, diffuse solar radiation and sunshine duration are measured on all these locations simultaneously. The data period and location for each station are shown in Table 1. Also the positions of these stations are shown in Fig. 1.

**Table 1. The location and data period of the main four radiation stations.**

No.	Station	Lat.	Lon.	Alt.	Data period
337	El-Arish	31.08	33.81	32	Apr 1998 - May 2000
464	Hurghada	27.28	33.75	6.5	Jan 1994 - Apr 1996
403	South Valley	26.18	32.7	77.3	Apr 2000 - Sep 2001
414	Aswan	23.97	32.78	192	Aug 1998 - Dec 2000

### B. Daily Global and UV-B Radiation Data

The period of UV-B radiation measurements is too short for statistical analysis in a climatological sense. However, a trial to describe some characteristics of UV-B radiation data available for about 30 months will be done. These data records of simultaneous measurements of daily UV-B radiation and total global solar irradiation have been analyzed to determine whether any systematic relationship exists between the two variables. The analysis of daily values of global solar radiation in (Mj/sq. m) and UV-B radiation in (Mj/sq. m) on all their locations are presented.

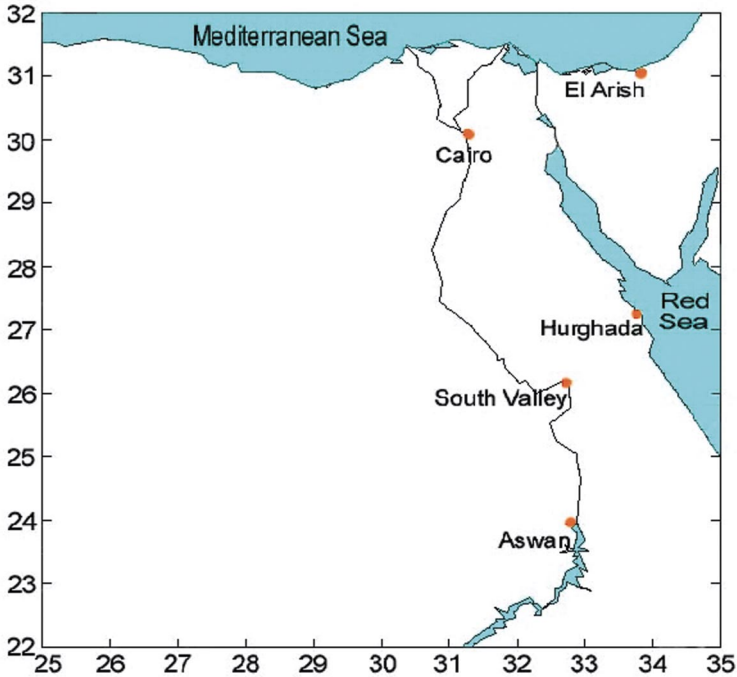


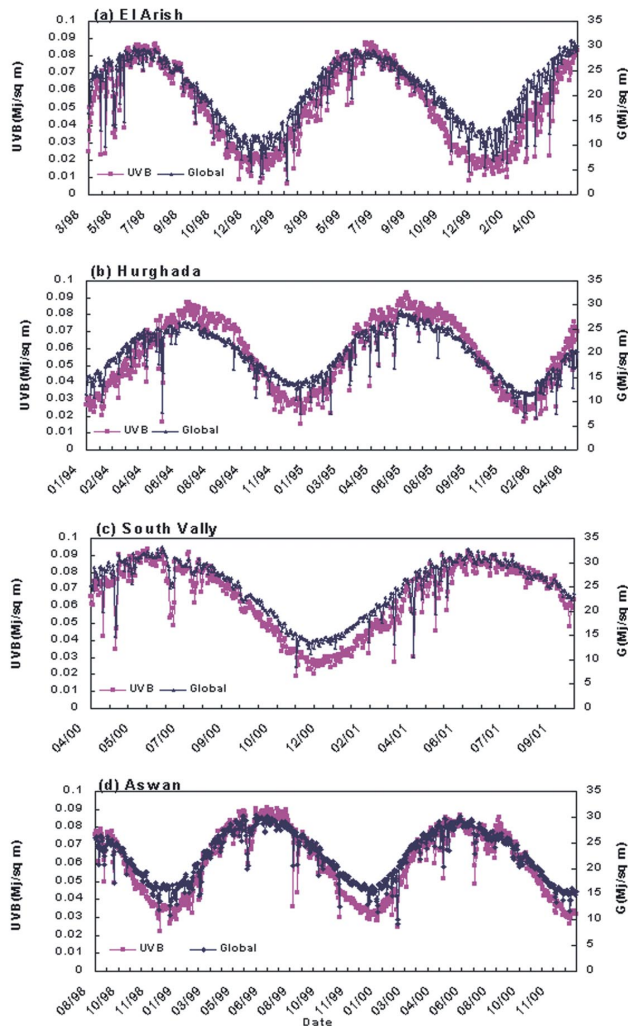
Fig. 1. The Egyptian chart with four radiation stations.

The daily-accumulated values of global solar radiation and UV-B radiation for El Arish are presented in Fig. 2a. It shows that the maximum values of global and UV-B radiation occurs in the summer months with the greatest value at July. The minimum values occur at winter with minimum values at January. It is clear that the annual wave is the dominant wave for the variation of both global and UV-B radiation. Also, it is clear that there is a high variability of the daily values of global and UV-B radiation during the year where it is influenced by cloud variability throughout the year.

Figure 2b shows the accumulated values of global and UV-B radiation for Hurghada during the period from January 1994 to April 1996. It shows that the maximum values also occur at the summer season while the minimum values occur at the winter. The maximum variability of the values of global and UV-B radiation occurs during the spring and winter seasons as a result of a high change in cloud cover over the station during these seasons.

Figure 2c illustrates the accumulated values of global and UV-B radiation of South Valley during the available period of study (April 2000 to September 2001). Similar to the first two stations, the maximum values occur during the summer while the minimum values occur during the winter. The measurements

of solar radiation at this station are influenced by two dominant factors, the sand and dust storms especially during the spring and summer seasons. The values of global and UV-B radiation for Aswan station during the period (August 1998 to December 2000) are presented in Fig. 2d. The influence of sand and dust storms during the spring and the beginning of summer season can be seen in the measurements of global and UV-B radiation. Also there are some cases of rising sand that occur during the winter as a characteristic of Aswan weather.



**Fig. 2.** Daily total UV-B radiation and global radiation measured at: (a) El-Arish from Apr 1998 to May 2000. (b) Hurghada from Jan1994 to Apr 1996. (c) South Vally from Apr 2000 to Sep 2001. (d) Aswan from Aug1998 to Dec 2000.

### C. Methodology

In this section, a theoretical method to calculate daily values of UV-B radiation based on global radiation is presented. The dimensionless values of global solar radiation ( $G^*$ ) and UV-B radiation ( $UVB^*$ ) can be written as (Hussain, 1992).

$$UVB^*(i) = \frac{UVB(i) - UVB_{\min}}{UVB_{\max} - UVB_{\min}} \quad (1)$$

$$G^*(i) = \frac{G(i) - G_{\min}}{G_{\max} - G_{\min}} \quad (2)$$

Where,  $i$  changes from 0 to  $N$ , the subscript “min” and “max” indicates the minimum and maximum values of UV-B radiation ( $UVB$ ) and global solar radiation ( $G$ ) for a complete year. The daily values of UV-B radiation and  $G$  are represented graphically during the available period of the stations El-Arish (Apr. 1998 to May 2000), Hurghada (Jan. 1994 to Apr. 1996) South Valley (Apr. 2000 to Sep. 2001) and Aswan (Aug. 1998 to Dec. 2000), in Fig. 2. The graph indicates the periodicity of the variations of the values of UV-B radiation and  $G$  at the selected stations. Therefore the dimensionless values of UV-B radiation and global solar radiation could be expanded in Fourier series as follows;

$$UVB^*(i) = \frac{A_{0UVB}}{2} + \sum_{j=1}^{\infty} \left[ A_{UVB j} \cos \frac{\pi j i}{m} + B_{UVB j} \sin \frac{\pi j i}{m} \right] \quad (3)$$

$$G^*(i) = \frac{A_{0G}}{2} + \sum_{j=1}^{\infty} \left[ A_{G j} \cos \frac{\pi j i}{m} + B_{G j} \sin \frac{\pi j i}{m} \right] \quad (4)$$

Where,  $i$  changes from 0 to  $N$ ,  $N$  is the number of days (record length), and  $m = N/2$ .

Subtracting equation (4) from equation (3), we obtain

$$UVB^*(i) - G^*(i) = \frac{A_0}{2} + \sum_{j=1}^{\infty} \left[ A_j \cos \frac{\pi j i}{m} + B_j \sin \frac{\pi j i}{m} \right] \quad (5)$$

Where,

$$A_0 = \frac{A_{0UVB} - A_{0G}}{2} \quad (6)$$

After simple mathematical treatment, the following expression for the Fourier coefficients can be obtained:

$$A_j = A_{UVB j} - A_{G j} = \frac{2}{N} \sum_{i=1}^N (UVB^*(i) - G^*(i)) \cos \frac{\pi j i}{m}, \quad (j=0,1,2,\dots) \quad (7)$$

and

$$B_j = B_{UVB_j} - B_{G_j} = \frac{2}{N} \sum_{i=1}^N (UVB^*(i) - G^*(i)) \sin \frac{\pi j i}{m}, (j=1,2,3,\dots) \quad (8)$$

Where, the summation in these equations extends over the number of observations (N). From (1), we can write (5) as

$$UVB(i) = UVB_{\min} + (UVB_{\max} - UVB_{\min}) \left( G^*(i) + \frac{A_0}{2} + \sum_{j=1}^{\infty} \left[ A_j \cos \frac{\pi j i}{m} + B_j \sin \frac{\pi j i}{m} \right] \right) \quad (9)$$

Kenisarin and Tkachenkova, 1992 found that for the number of observations N, the number of harmonics that describes the variation of the periodic function (UVB – G) sufficiently will equal N/2. Since we are dealing with the daily sum values of UV-B radiation and global solar radiation, then the sum in (9) of the Fourier series can be written as;

$$UVB(i) = UVB_{\min} + (UVB_{\max} - UVB_{\min}) \left( G^*(i) + \frac{A_0}{2} + \sum_{j=1}^{\infty} \left[ A_j \cos \frac{\pi j i}{m} + B_j \sin \frac{\pi j i}{m} \right] \right) \quad (10)$$

### 3. Results

The available data of global and UV-B radiation of the four stations (Aswan, South Valley, Hurghada, El-Arish) have been employed together with equations 7 and 8 to determine the Fourier coefficients for these stations. Fourier coefficients were used to calculate the UV-B radiation for the same reference stations by using equation 10. Figure 3 illustrates the error between the observed and calculated values of UV-B radiation at the four stations. Also Table 2 shows the root mean square error (RMSE) and mean absolute error (MAER) between the observed and calculated values of UV-B radiation for the four stations. It is clear that there is close agreement between the measured and calculated values of UV-B radiation at the four stations with the smallest error at Hurghada.

**Table 2. The root mean square error (RMSE) and mean absolute error (MAER) between the observed and calculated values of UV-B radiation.**

Station	RMSE	MAER
El- Arish	0.00079	0.00029
Hurghada	0.00026	0.00004
South Valley	0.00028	0.00022

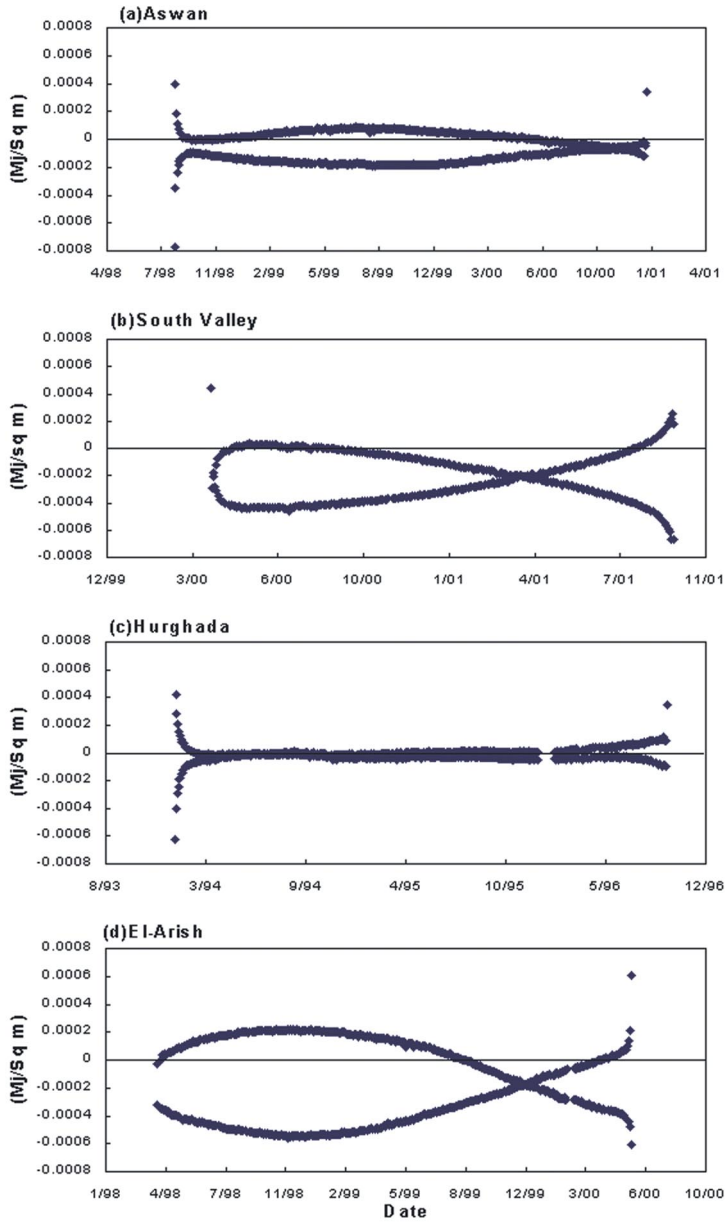


Fig. 3. The error between the observation and estimates using Fourier series at: (a) Aswan, (b) South Valley, (c) Hurghada, (d) El-Arish.

#### 4. Verification Example

In order to verify the method, two stations were chosen (South Valley and Rafah) each of these having measurements of global solar radiation and lied in neighboring of one of the above station. Fourier coefficients for the global radiation of Aswan and El-Arish have been used to calculate the UV-B radiation at South Valley and Rafah stations respectively. Table 3 shows the RMSE and mean absolute error between the observed and calculated values of UV-B radiation at South Valley and Rafah stations respectively. Comparing the order of RMSE and MAER values in Table 3 with that in Table 2, the values of RMSE and MAER in Table 3 are higher than those in Table 2. In other words, it can be said that the best application for this method is on the same location of the measurements.

**Table 3. The root mean square error (RMSE) and mean absolute error (MAER) between the observed and calculated values of UVB radiation at Rafah and South Valley stations.**

Station	RMSE	MAER
Rafah	0.010274	0.009173
South valley	0.010726	0.009346

#### 5. Conclusion

The analysis of the observed values of global and UV-B radiation at four stations illustrates that there is a good relation between the two variables and a periodicity of the variations in the two variables throughout the available period of study of each station. This good relationship makes us able to obtain a mathematical relationship between the two variables. Because of the periodicity of variations in global and UV-B radiation, their non-dimensional values are expanded in Fourier series. Fourier coefficients were determined by using measured global solar radiation data of the selected Egyptian weather stations. These coefficients were used to calculate UV-B radiation based on global solar radiation for the same stations. A comparison between the observed and calculated UV-B radiation shows a good agreement between the observed and estimated UV-B radiation. The method can also be applied to calculate the UV-B radiation for other neighboring stations where UV-B radiation measurements are unavailable. This can be done by using records of global solar radiation at these stations. The method is an excellent technique, which can be used when the required meteorological data are not available.



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## تقدير الأشعة فوق البنفسجية - ب بواسطة متسلسلات فورير

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المستخلص. إن معرفة وتقدير القيم اليومية للأشعة فوق البنفسجية- ب يعتبر مهما لتقديم معلومات للناس عن التأثيرات الضارة على صحة الإنسان من الأشعة فوق البنفسجية-ب. والهدف الرئيسي من هذا البحث هو إيجاد علاقة بين الأشعة فوق البنفسجية- ب والإشعاع الكلي. وبما أن التغيرات في قيم الإشعاع الكلي و الأشعة فوق البنفسجية- ب تعتبر دورية، فإن القيم غير البعدية يمكن تمديدها في متسلسلات فورير. وقد تم حساب معاملات فورير باستخدام البيانات المقاسة للإشعاع الكلي لبعض محطات مصر. ومن ثم استخدمت هذه المعاملات لحساب الأشعة فوق البنفسجية- ب باستخدام الإشعاع الشمسي الكلي المقاس عند هذه المحطات. وقد تم مقارنة القيم المقاسة والمحسوبة للأشعة فوق البنفسجية- ب وعرض النتائج في صورة أشكال للفروق. وأظهرت المقارنة أن هناك توافقاً جيداً بين القيم المقاسة والمحسوبة للأشعة فوق البنفسجية- ب. وقد طبقت الطريقة أيضاً لحساب الأشعة فوق البنفسجية- ب ، لمحطات لا تقاس بها الأشعة فوق البنفسجية- ب ، وذلك باستخدام قيم الإشعاع الشمسي الكلي لهذه المحطات.