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Dear Dr. MOHAMMED SH. ALHUTHALI

I am very pleased to inform you that your manuscript entitle :

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Solitary Wave Solutions for the Nonlinear Ito System

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ABSTRACT. In this paper, we look for some new solitary wave solutions for the nonlinear Ito coupled system. For this purpose, we improved the extended tanh method to construct more new exact doubly-periodic solutions in terms of tanh and Jacobi elliptic functions of nonlinear Ito system.The solitary wave solutions and triangular periodic solutions also can be obtained .

Keywords: Coupled evolution system; Weierslrass and Jacobi elliptic functions; Travelling wave solution; Soliton solution; Periodic solution.

1. Introduction

In recent years, directly searching for exact solutions of nonlinear partial differential equations has become increasingly attractive, partly due to the availability of computer symbolic systems like Maple or Mathematica ,which allow us to perform complicated and tedious algebraic calculations, as well as help us to find exact solutions of nonlinear partial differential equations. It is well known that the nonlinear partial differential equations are widely used to describe many important phenomena in physics, biology, chemistry, *etc.* The nonlinear Ito system plays a crucial rule in applied mathematics and physics and has many applications in physics and engineering [1,2]. The travelling wave exact solution of nonlinear partial differential equations have been studied extensively, see for example [1,5] . Many authors presented various powerful methods to deal with this subject. The main purpose of these studies is to construct the solitary wave solutions and periodic solutions as a polynomial in hyperbolic and triangle functions[5-11].

2. Summary of the generalized tanh method

In this section , we investigate the exact travelling wave solutions for the following Nonlinear Ito coupled system:

$$\begin{aligned} &u_t = v_x, \\ &v_t = -2(v_{xxx} + 3uv_x + 3vu_x) - 12ww_x, \\ &w_t = w_{xxx} + 3uw_x \end{aligned} \tag{1}$$

By the travelling transformation

$$u = u(\zeta), v = v(\zeta), w = w(\zeta), \zeta = x + ct \tag{2}$$

where c is constant, the Eq.1 becomes

$$\begin{aligned} &cu' - v' = 0 \\ &cv' + 2v''' + 6uv' + 6vu' + 12ww' = 0 \\ &cw' - w''' - 3uw' = 0 \end{aligned} \tag{3}$$

Suppose that

$$\begin{array}{l} u(\zeta) = \sum_{i=0}^{n_1} a_i\varphi^i + \sum_{i=1}^{n_1} \frac{b_i}{\varphi^i} \\ v(\zeta) = \sum_{i=0}^{n_2} g_i\varphi^i + \sum_{i=1}^{n_2} \frac{d_i}{\varphi^i} \\ w(\zeta) = \sum_{i=0}^{n_3} h_i\varphi^i + \sum_{i=1}^{n_3} \frac{k_i}{\varphi^i} \end{array} \tag{4}$$

where $\varphi(\zeta)$ satisfies

$$\varphi' = \epsilon\sqrt{\sum_{j=0}^{r} c_i\varphi^i} \tag{5}$$

Where $\epsilon = \pm 1$.Balancing the highest derivatives term with the nonlinear term in Eq. (3) gives $r = n_1 + 2, n_1 = n_2 \geq n_3$.By choosing $r = 4, n_1 = n_2 = n_3 = 2$, we have

$$\begin{array}{l} u = a_0 + a_1\varphi + a_2\varphi^2 + \frac{b_1}{\varphi} + \frac{b_2}{\varphi^2} \\ v = g_0 + g_1\varphi + g_2\varphi^2 + \frac{d_1}{\varphi} + \frac{d_2}{\varphi^2} \\ w = h_0 + h_1\varphi + h_2\varphi^2 + \frac{k_1}{\varphi} + \frac{k_2}{\varphi^2} \end{array} \tag{6}$$

and

$$\varphi' = \epsilon\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4} \tag{7}$$

By substituting (6) and (7) into (3) ,collecting coefficients oflike terms and set them equal zero , we get a set of over-determined algebraic equations with respect to $a_0, a_1, a_2, b_1, b_2, g_0, g_1, g_2, d_1, d_2, h_0, h_1, h_2, k_1, k_2, c_0, c_1, c_2, c_3$ and c_4.With the aid of maple and Wu-elimination[8], we find the following results.

Case 1. $\{a_1 = a_2 = g_1 = g_2 = h_1 = h_2 = k_2 = 0, c_1 = c_1, c_2 = c_2, c_3 = c_3, c_4 = c_4,$
$k_1 = k_1, a_0 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c, b_1 = -\epsilon^2 c_1, b_2 = -2\epsilon^2 c_0,$
$$g_0 = -\frac{-\epsilon^4 c_1^2 c + 2c^2\epsilon^2 c_0 + 4\epsilon^4 c c_2 c_0 - 2k_1^2}{4\epsilon^2 c_0}, d_1 = -\epsilon^2 c c_1, d_2 = -2\epsilon^2 c c_0,$$
$$h_0 = -\frac{-\epsilon^4 c_1^3 c + 4c\epsilon^4 c_0 c_1 c_2 - 2c_1 k_1^2 - 8\epsilon^4 c_0^2 c c_3}{8c_0 k_1}\}$$

In this case φ'will taken the form
$\varphi' = \epsilon\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}$
and solitary wave solutions take the form :
(1) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 < 0, c_4 > 0$
$$u_1 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c + \frac{c_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$
$$v_1 = -\frac{(\frac{c^2\epsilon^2 c_2^2}{2c_4} + \frac{\epsilon^4 c_2^3 c}{c_4} - 2k_1^2)c_4}{\epsilon^2 c_2^2} + \frac{cc_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$
$$w_1 = \frac{2k_1}{\epsilon\sqrt{-\frac{2c_2}{c_4}}\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)}.$$

(2) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 > 0, c_4 > 0$

$u_2 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c - \dfrac{c_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}$,

$v_2 = -\dfrac{(\dfrac{c^2\epsilon^2 c_2^2}{2c_4} + \dfrac{\epsilon^4 c_2^3 c}{c_4} - 2k_1^2)c_4}{\epsilon^2 c_2^2} - \dfrac{cc_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}$,

$w_2 = \dfrac{2k_1}{\epsilon\sqrt{\dfrac{2c_2}{c_4}}\tan(\frac{1}{2}\sqrt{2c_2}\zeta)}$.

Case 2. $\{b_1 = b_2 = d_1 = d_2 = h_2 = k_1 = k_2 = 0, c_1 = c_1, c_2 = c_2, c_3 = c_3, c_4 = c_4,$
$h_1 = h_1, a_0 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c, a_1 = -\epsilon^2 c_3, a_2 = -2\epsilon^2 c_4,$
$g_0 = -\dfrac{-\epsilon^4 c_3^2 c + 2c^2\epsilon^2 c_4 + 4\epsilon^4 c c_2 c_4 - 2h_1^2}{4\epsilon^2 c_4}, g_1 = -\epsilon^2 c c_3, g_2 = -2\epsilon^2 c c_4,$
$h_0 = -\dfrac{-\epsilon^4 c_3^3 c + 4c\epsilon^4 c_2 c_3 c_4 - 2c_3 h_1^2 - 8\epsilon^4 c_4^2 c c_1}{8c_4 h_1}\}$

Then φ'will be
$\varphi' = \epsilon\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}$
The solitary wave solutions take the form of eliptic and Jacobic elliptic function:
(1) If $c_0 = c_1 = c_3 = 0, c_2 > 0, c_4 < 0$

$u_6 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c + 2\epsilon^2 c_2 \sec h(\sqrt{c_2}\zeta)^2$,

$v_6 = -\dfrac{4\epsilon^4 c c_2 c_4 + 2c^2\epsilon^2 c_4 - 2h_1^2}{4\epsilon^2 c_4} + 2\epsilon^2 c c_2 \sec h(\sqrt{c_2}\zeta)^2$,

$w_6 = h_1\sqrt{-\dfrac{c_2}{c_4}}\sec h(\sqrt{c_2}\zeta)$.

(2) If $c_0 = c_1 = c_3 = 0, c_2 < 0, c_4 > 0$

$u_7 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c + 2\epsilon^2 c_2 \sec(\sqrt{-c_2}\zeta)^2$,

$v_7 = -\dfrac{4\epsilon^4 c c_2 c_4 + 2c^2\epsilon^2 c_4 - 2h_1^2}{4\epsilon^2 c_4} + 2\epsilon^2 c c_2 \sec(\sqrt{-c_2}\zeta)^2$,

$w_7 = h_1\sqrt{-\dfrac{c_2}{c_4}}\sec(\sqrt{-c_2}\zeta)$.

(3) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 < 0, c_4 > 0$

$u_9 = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c + \epsilon^4 c_2 \tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2$,

$v_9 = -\dfrac{4\epsilon^4 c c_2 c_4 + 2c^2\epsilon^2 c_4 - 2h_1^2}{4\epsilon^2 c_4} + \epsilon^4 c c_2 \tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2$,

$w_9 = \frac{1}{2}h_1\epsilon\sqrt{-\dfrac{2c_2}{c_4}}\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)$.

(4) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 > 0, c_4 > 0$

$u_{10} = -\frac{1}{3}\epsilon^2 c_2 + \frac{1}{3}c - \epsilon^4 c_2 \tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2$,

$v_{10} = -\dfrac{4\epsilon^4 c c_2 c_4 + 2c^2\epsilon^2 c_4 - 2h_1^2}{4\epsilon^2 c_4} - \epsilon^4 c c_2 \tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2$,

$w_{10} = \frac{1}{2}h_1\epsilon\sqrt{\dfrac{2c_2}{c_4}}\tan(\frac{1}{2}\sqrt{2c_2}\zeta)$.

(5) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2 m^2(1-m^2)}{c_4(2m^2-1)^2}, c_2 > 0, c_4 < 0$

$$u_{11} = -\tfrac{1}{3}\epsilon^2 c_2 + \tfrac{1}{3}c + \frac{2\epsilon^2 c_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1},$$

$$v_{11} = -\frac{4\epsilon^4 cc_2c_4 + 2c^2\epsilon^2 c_4 - 2h_1^2}{4\epsilon^2 c_4} + \frac{2\epsilon^2 cc_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1},$$

$$w_{11} = h_1\sqrt{-\frac{c_2 m^2}{c_4(2m^2-1)}}cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta).$$

(6) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2(1-m^2)}{c_4(2-m^2)^2}, c_2 > 0, c_4 < 0$

$$u_{12} = -\tfrac{1}{3}\epsilon^2 c_2 + \tfrac{1}{3}c + \frac{2\epsilon^2 c_2 dn(\sqrt{\frac{c_2}{2-m^2}}\zeta)^2}{2-m^2},$$

$$v_{12} = -\frac{4\epsilon^4 cc_2c_4 + 2c^2\epsilon^2 c_4 - 2h_1^2}{4\epsilon^2 c_4} + \frac{2\epsilon^2 cc_2 dn(\sqrt{\frac{c_2}{2-m^2}}\zeta)^2}{2-m^2},$$

$$w_{12} = h_1\sqrt{-\frac{c_2}{c_4(2-m^2)}}dn(\sqrt{\frac{c_2}{2-m^2}}\zeta).$$

Case 3. $\{k_2 = 0, b_2 = 0, h_0 = \dfrac{2g_0 + c^2}{4\sqrt{-\frac{1}{2}c}}, g_0 = g_0, c_2 = c_2, c_3 = c_3, c_1 = c_1, a_2 = 0,$
$g_1 = 0, g_2 = 0, d_2 = 0, a_1 = 0, h_1 = 0, h_2 = 0, a_0 = \tfrac{1}{3}c - \tfrac{1}{3}\epsilon^2 c_2, c_4 = 0, b_1 = -\epsilon^2 c_1,$
$d_1 = -\epsilon^2 cc_1, k_1 = \sqrt{-\tfrac{1}{2}c\epsilon^2 c_1}, c_0 = 0\}$.

Then φ'will be
$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + c_3\varphi^3}$
The solitary wave solution take the form when $c_2 =$

$$u_{15} = \tfrac{1}{3}c - \frac{\epsilon^2 c_1 c_3}{\sqrt{-c_1c_3}\left((sn(\frac{1}{2}\sqrt{2}(-c_1c_3)^{(\frac{1}{4})}\epsilon\zeta, \frac{1}{2}\sqrt{2})^2) - 1\right)},$$

$$v_{15} = g_0 - \frac{\epsilon^2 cc_1 c_3}{\sqrt{-c_1c_3}\left((sn(\frac{1}{2}\sqrt{2}(-c_1c_3)^{(\frac{1}{4})}\epsilon\zeta, \frac{1}{2}\sqrt{2})^2) - 1\right)},$$

$$w_{15} = \frac{2g_0 + c^2}{4\sqrt{-\frac{1}{2}c}} + \frac{\sqrt{-\frac{1}{2}}\epsilon^2 cc_1c_3}{\sqrt{-c_1c_3}\left((sn(\frac{1}{2}\sqrt{2}(-c_1c_3)^{(\frac{1}{4})}\epsilon\zeta, \frac{1}{2}\sqrt{2})^2) - 1\right)}.$$

Case 4. $\{c_0 = c_0, k_2 = 0, a_0 = a_0, h_0 = h_0, c_2 = c_2, c_1 = c_1, a_2 = 0, g_1 = 0, g_2 = 0,$
$a_1 = 0, h_1 = 0, h_2 = 0, k_1 = 0, c_4 = 0, b_1 = -\epsilon^2 c_1, b_2 = -2\epsilon^2 c_0, d_2 = -2\epsilon^2 cc_0,$
$g_0 = -\dfrac{c(12a_0c_0 - 3c_1^2\epsilon^2 + 2cc_0 + 16\epsilon^2 c_0c_2)}{12c_0}, d_1 = -\epsilon^2 cc_1, c_3 = -\dfrac{c_1(c_1^2 - 4c_0c_2)}{8c_0^2}\}$

Then φ'will be

$\varphi' = \epsilon\sqrt{c_0 + c_1\varphi + c_2\varphi^2 - \frac{c_1(c_1^2-4c_0c_2)}{8c_0^2}\varphi^3}$

The solitary wave solutions take the form :
(1) If $c_1 = 0, c_2 < 0$

$$u_{16} = a_0 - \frac{2\epsilon^2 c_2}{\sinh(\sqrt{c_2}\epsilon\zeta)^2},$$
$$v_{16} = -\frac{c(12a_0c_0 + 2cc_0 + 16\epsilon^2 c_0c_2)}{12c_0} - \frac{2\epsilon^2 cc_2}{\sinh(\sqrt{c_2}\epsilon\zeta)^2},$$
$$w_{16} = h_0.$$

(2) If $c_1 = 0, c_2 > 0$

$$u_{17} = a_0 + \frac{2\epsilon^2 c_2}{\sin(\sqrt{-c_2}\epsilon\zeta)^2},$$
$$v_{17} = -\frac{c(12a_0c_0 + 2cc_0 + 16\epsilon^2 c_0c_2)}{12c_0} + \frac{2\epsilon^2 cc_2}{\sin(\sqrt{-c_2}\epsilon\zeta)^2},$$
$$w_{17} = h_0.$$

Case 5. $\{c_0 = c_0, k_1 = k_1, k_2 = 0, c_2 = c_2, c_3 = c_3, c_1 = c_1, a_2 = 0, g_1 = 0, g_2 = 0,$
$a_1 = 0, h_1 = 0, h_2 = 0, a_0 = \frac{1}{3}c - \frac{1}{3}\epsilon^2 c_2, c_4 = 0, b_1 = -\epsilon^2 c_1, d_1 = -\epsilon^2 cc_1,$
$b_2 = -2\epsilon^2 c_0, d_2 = -2\epsilon^2 cc_0, g_0 = -\dfrac{2c^2\epsilon^2 c_0 + 4\epsilon^4 cc_0c_2 - \epsilon^4 cc_1^2 - 2k_1^2}{4\epsilon^2 c_0},$
$h_0 = \dfrac{-4\epsilon^4 cc_0c_1c_2 + \epsilon^4 cc_1^3 + 2c_1k_1^2 + 8\epsilon^4 cc_0^2c_3}{8c_0k_1}\}$

Then φ'will be

$\varphi' = \epsilon\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3}$
The solitary wave solutions take the form :
(1) If $c_3 = 0, c_0 = \dfrac{c_1^2}{4c_2}$

$$u_{18} = -\tfrac{1}{3}\epsilon^2 c_2 + \tfrac{1}{3}c - \frac{2\epsilon^2 c_1c_2}{2e^{(\sqrt{c_2}\epsilon\zeta)}c_2 - c_1} - \frac{2\epsilon^2 c_1^2c_2}{(2e^{(\sqrt{c_2}\epsilon\zeta)}c_2 - c_1)^2},$$
$$v_{18} = -\frac{\left(\dfrac{\epsilon^2 cc_1^2}{2c_2} - 2k_1^2\right)c_2}{\epsilon^2 c_1^2} - \frac{2\epsilon^2 c_1cc_2}{2e^{(\sqrt{c_2}\epsilon\zeta)}c_2 - c_1} - \frac{2\epsilon^2 c_1^2cc_2}{(2e^{(\sqrt{c_2}\epsilon\zeta)}c_2 - c_1)^2},$$
$$w_{18} = \frac{k_1c_2}{c_1} + \frac{2k_1c_2}{2e^{(\sqrt{c_2}\epsilon\zeta)}c_2 - c_1}.$$

Case 6. $\{c_0 = c_0, k_1 = k_1, h_0 = \dfrac{2g_0 + c^2}{2\sqrt{-2c}}, g_0 = g_0, c_2 = c_2, a_2 = 0, g_1 = 0,$
$g_2 = 0, a_1 = 0, h_1 = 0, h_2 = 0, b_1 = -\dfrac{2k_1}{\sqrt{-2c}}, d_1 = k_1\sqrt{-2c}, c_4 = 0,$
$c_1 = \dfrac{k_1}{\sqrt{-2c}\epsilon^2}, k_2 = 2\sqrt{-2c}\epsilon^2 c_0, b_2 = -4\epsilon^2 c_0, d_2 = -4\epsilon^2 cc_0,$
$a_0 = \dfrac{-3k_1^2 + 8c^2\epsilon^2 c_0 - 32\epsilon^4 cc_0c_2}{24\epsilon^2 cc_0}, c_3 = \dfrac{(k_1^2 + 8\epsilon^4 cc_0c_2)k_1}{16c_0^2\sqrt{-2c}\epsilon^6 c}\}$

Then φ'will be

$$\varphi' = \epsilon\sqrt{c_0 + \frac{k_1}{\sqrt{-2c\epsilon^2}}\varphi + c_2\varphi^2 + \frac{(k_1^2 + 8\epsilon^4 cc_0c_2)k_1}{16c_0^2\sqrt{-2c\epsilon^6 c}}\varphi^3}$$

The solitary wave solutions take the form :

(1) If $k_1 = 0, c_2 > 0$

$$u_{20} = \frac{8c^2\epsilon^2c_0 - 32\epsilon^4cc_0c_2}{24\epsilon^2cc_0} - \frac{4\epsilon^2c_2}{\sinh(\sqrt{c_2}\epsilon\zeta)^2},$$

$$v_{20} = g_0 - \frac{4\epsilon^2cc_2}{\sinh(\sqrt{c_2}\epsilon\zeta)^2},$$

$$w_{20} = \tfrac{2g_0+c^2}{2\sqrt{-2c}} + \frac{2\sqrt{-2c}\epsilon^2c_2}{\sinh(\sqrt{c_2}\epsilon\zeta)^2}.$$

(2) If $k_1 = 0, c_2 < 0$

$$u_{21} = \frac{8c^2\epsilon^2c_0 - 32\epsilon^4cc_0c_2}{24\epsilon^2cc_0} + \frac{4\epsilon^2c_2}{\sin(\sqrt{-c_2}\epsilon\zeta)^2},$$

$$v_{21} = g_0 + \frac{4\epsilon^2cc_2}{\sin(\sqrt{-c_2}\epsilon\zeta)^2},$$

$$w_{21} = \frac{2g_0+c^2}{2\sqrt{-2c}} - \frac{2\sqrt{-2c}\epsilon^2c_2}{\sin(\sqrt{-c_2}\epsilon\zeta)^2}.$$

(3) If $k_1 = 2\sqrt{-2c_2cc_0}\epsilon^2$

$$u_{22} = \frac{-8\epsilon^4cc_0c_2 + 8\epsilon^2c^2c_0}{24\epsilon^2cc_0} - \frac{4\sqrt{-2c_2cc_0}\epsilon^2c_2\sqrt{-c}}{\sqrt{-2c}(e^{\sqrt{c_2}\epsilon\zeta}c_2\sqrt{-c} - \sqrt{-c_2cc_0})} + \frac{4\epsilon^2cc_2^2c_0}{(e^{\sqrt{c_2}\epsilon\zeta}c_2\sqrt{-c} - \sqrt{-c_2cc_0})^2},$$

$$v_{22} = g_0 + \frac{2\sqrt{-2c_2cc_0}\epsilon^2\sqrt{-2c}c_2\sqrt{-c}}{(e^{\sqrt{c_2}\epsilon\zeta}c_2\sqrt{-c} - \sqrt{-c_2cc_0})} + \frac{4\epsilon^2c^2c_2^2c_0}{(e^{\sqrt{c_2}\epsilon\zeta}c_2\sqrt{-c} - \sqrt{-c_2cc_0})^2},$$

$$w_{22} = \frac{2g_0+c^2}{2\sqrt{-2c}} + \frac{2\sqrt{-2c_2cc_0}\epsilon^2c_2\sqrt{-c}}{(e^{\sqrt{c_2}\epsilon\zeta}c_2\sqrt{-c} - \sqrt{-c_2cc_0})} - \frac{2\sqrt{-2c}\epsilon^2cc_2^2c_0}{(e^{\sqrt{c_2}\epsilon\zeta}c_2\sqrt{-c} - \sqrt{-c_2cc_0})^2}.$$

Case 7. $\{k_2 = 0, b_2 = 0, a_0 = a_0, h_0 = h_0, c_2 = c_2, c_3 = c_3, c_1 = c_1, a_2 = 0, b_1 = 0, g_2 = 0, d_1 = 0, d_2 = 0, h_1 = 0, h_2 = 0, k_1 = 0, g_0 = -\frac{1}{3}\epsilon^2c_2c - a_0c - \frac{1}{6}c^2, g_1 = -\frac{1}{2}\epsilon^2cc_3, a_1 = -\frac{1}{2}\epsilon^2c_3, c_4 = 0, c_0 = 0\}$.

Then φ'will be

$$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + c_3\varphi^3}$$

The solitary wave solutions take the form :

(1) If $c_1 = 0, c_2 > 0$

$$u_{23} = a_0 + \tfrac{1}{2}\epsilon^2c_2\sec h(\tfrac{1}{2}\sqrt{c_2}\zeta)^2,$$
$$v_{23} = -\tfrac{1}{3}\epsilon^2c_2c - a_0c - \tfrac{1}{6}c^2 + \tfrac{1}{2}\epsilon^2cc_2\sec h(\tfrac{1}{2}\sqrt{c_2}\zeta)^2,$$
$$w_{23} = h_0.$$

(2) If $c_1 = 0, c_2 < 0$

$$u_{24} = a_0 + \tfrac{1}{2}\epsilon^2c_2\sec(\tfrac{1}{2}\sqrt{-c_2}\zeta)^2,$$
$$v_{24} = -\tfrac{1}{3}\epsilon^2c_2c - a_0c - \tfrac{1}{6}c^2 + \tfrac{1}{2}\epsilon^2cc_2\sec(\tfrac{1}{2}\sqrt{-c_2}\zeta)^2,$$
$$w_{24} = h_0.$$

Case 8. $\{k_2 = 0, b_2 = 0, a_0 = a_0, h_0 = h_0, c_2 = c_2, c_3 = c_3, c_1 = c_1, a_2 = 0, g_2 = 0, d_2 = 0, h_1 = 0, h_2 = 0, k_1 = 0, g_0 = -\frac{1}{3}\epsilon^2c_2c - a_0c - \frac{1}{6}c^2, g_1 = -\frac{1}{2}\epsilon^2cc_3, a_1 = -\frac{1}{2}\epsilon^2c_3, c_4 = 0, b_1 = -\frac{1}{2}\epsilon^2c_1, d_1 = -\frac{1}{2}\epsilon^2cc_1, c_0 = 0\}$.

Then φ' will be
$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + c_3\varphi^3}$
The solitary wave solutions take the form :
(1) If $c_2 = 0$

$$u_{27} = a_0 - \tfrac{1}{2}\epsilon^2\sqrt{-c_1c_3}\left(sn(\tfrac{1}{2}\sqrt{2}(-c_1c_3)^{\frac{1}{4}}\epsilon\zeta, \tfrac{1}{2}\sqrt{2})^2 - 1\right) - \frac{\epsilon^2 c_1c_3}{2\sqrt{-c_1c_3}\left(sn(\tfrac{1}{2}\sqrt{2}(-c_1c_3)^{\frac{1}{4}}\epsilon\zeta, \tfrac{1}{2}\sqrt{2})^2 - 1\right)},$$

$$v_{27} = -a_0c - \tfrac{1}{6}c^2 - \tfrac{1}{2}\epsilon^2c\sqrt{-c_1c_3}\left(sn(\tfrac{1}{2}\sqrt{2}(-c_1c_3)^{\frac{1}{4}}\epsilon\zeta, \tfrac{1}{2}\sqrt{2})^2 - 1\right) - \frac{\epsilon^2 cc_1c_3}{2\sqrt{-c_1c_3}\left(sn(\tfrac{1}{2}\sqrt{2}(-c_1c_3)^{\frac{1}{4}}\epsilon\zeta, \tfrac{1}{2}\sqrt{2})^2 - 1\right)},$$

$w_{27} = h_0.$

Case 9. $\{k_2 = 0, b_2 = 0, g_0 = g_0, c_2 = c_2, c_3 = c_3, c_1 = c_1, a_2 = 0, b_1 = 0,$
$g_2 = 0, d_1 = 0, d_2 = 0, h_0 = \dfrac{2g_0 + c^2}{4\sqrt{-\frac{1}{2}c}}, h_2 = 0, k_1 = 0, a_0 = \tfrac{1}{3}c - \tfrac{1}{3}\epsilon^2c_2,$
$g_1 = -\epsilon^2cc_3, a_1 = -\epsilon^2c_3, h_1 = \sqrt{-\tfrac{1}{2}c\epsilon^2c_3}, c(4) = 0, c(0) = 0\}$

Then φ' will be
$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + c_3\varphi^3}$

The solitary wave solutions take the form :

(1) If $c_0 = 0, c_2 > 0$
$u_{33} = \frac{1}{3}c - \frac{1}{3}\epsilon^2c_2 + \epsilon^2c_2\sec h(\frac{1}{2}\sqrt{c_2}\zeta)^2,$
$v_{33} = g_0 + \epsilon^2cc_2\sec h(\frac{1}{2}\sqrt{c_2}\zeta)^2,$
$w_{33} = \frac{2g_0+c^2}{4\sqrt{-\frac{1}{2}c}} - \sqrt{-\frac{1}{2}cc_2\epsilon^2}\sec h(\frac{1}{2}\sqrt{c_2}\zeta)^2.$

(2) If $c_1 = 0, c_2 < 0$
$u_{34} = \frac{1}{3}c - \frac{1}{3}\epsilon^2c_2 + \epsilon^2c_2\sec(\frac{1}{2}\sqrt{-c_2}\zeta)^2,$
$v_{34} = g_0 + \epsilon^2cc_2\sec(\frac{1}{2}\sqrt{-c_2}\zeta)^2,$
$w_{34} = \frac{2g_0+c^2}{4\sqrt{-\frac{1}{2}c}} - \sqrt{-\frac{1}{2}cc_2\epsilon^2}\sec(\frac{1}{2}\sqrt{-c_2}\zeta)^2.$

Case 10. $\{k_2 = 0, b_2 = 0, h_1 = h_1, h_0 = \dfrac{2g_0 + c^2}{4\sqrt{-\frac{1}{2}c}}, g_0 = g_0, c_2 = c_2, c_1 = c_1,$
$a_2 = 0, g_2 = 0, d_2 = 0, g_1 = 2h_1\sqrt{-\tfrac{1}{2}c}, a_1 = \dfrac{2h_1\sqrt{-\frac{1}{2}c}}{c}, h_2 = 0,$
$a_0 = \tfrac{1}{3}c - \tfrac{1}{3}\epsilon^2c_2, c_3 = \dfrac{h_1}{\sqrt{-\frac{1}{2}c\epsilon^2}}, c_4 = 0, b_1 = -\epsilon^2c_1, d_1 = -\epsilon^2cc_1,$
$k_1 = \sqrt{-\tfrac{1}{2}c\epsilon^2c_1}, c(0) = 0\}.$

Then φ' will be
$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + \frac{h_1}{\sqrt{-\frac{1}{2}c\epsilon^2}}\varphi^3}$
The solitary wave solutions take the form :

(1) If $c_2 = 0$

$$u_{37} = \frac{1}{3}c + \sqrt{-\frac{1}{2}c\epsilon}\sqrt{-2h_1c_1\sqrt{-2c}}\left(sn(\frac{\sqrt{-c\sqrt{-2c}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}}}{2c}\zeta, \frac{1}{2}\sqrt{2})^2 - 1\right)$$
$$-\frac{2\epsilon c_1 h_1}{\sqrt{-2h_1c_1\sqrt{-2c}}\left(sn(\frac{\sqrt{-c\sqrt{-2c}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}}}{2c}\zeta, \frac{1}{2}\sqrt{2})^2 - 1\right)},$$

$$v_{37} = g_0 + \sqrt{-\frac{1}{2}c\epsilon}\sqrt{-2h_1c_1\sqrt{-2c}}\left(sn(\frac{\sqrt{-c\sqrt{-2c}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}}}{2c}\zeta, \frac{1}{2}\sqrt{2})^2 - 1\right)$$
$$-\frac{2\epsilon c c_1 h_1}{\sqrt{-2h_1c_1\sqrt{-2c}}\left(sn(\frac{\sqrt{-c\sqrt{-2c}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}}}{2c}\zeta, \frac{1}{2}\sqrt{2})^2 - 1\right)},$$

$$w_{37} = \frac{2g_0 + c^2}{4\sqrt{-\frac{1}{2}c}} + \frac{1}{2}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}\left(sn(\frac{\sqrt{-c\sqrt{-2c}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}}}{2c}\zeta, \frac{1}{2}\sqrt{2})^2 - 1\right)$$
$$+\frac{2\sqrt{-\frac{1}{2}c}\epsilon c_1 h_1}{\sqrt{-2h_1c_1\sqrt{-2c}}\left(sn(\frac{\sqrt{-c\sqrt{-2c}\epsilon\sqrt{-2h_1c_1\sqrt{-2c}}}}{2c}\zeta, \frac{1}{2}\sqrt{2})^2 - 1\right)}.$$

Case 11. $\{k_2 = 0, b_2 = 0, a_0 = a_0, c_4 = c_4, h_0 = h_0, c_2 = c_2, c_3 = c_3, c_1 = c_1,$
$a_2 = 0, g_1 = 0, g_2 = 0, d_2 = 0, a_1 = 0, h_1 = 0, h_2 = 0, k_1 = 0, b_1 = -\frac{1}{2}\epsilon^2 c_1,$
$g_0 = -\frac{1}{3}\epsilon^2 c_2 c - a_0 c - \frac{1}{6}c^2, d_1 = -\frac{1}{2}\epsilon^2 c c_1, c_0 = 0\}$

Then φ' will be
$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}$
The solitary wave solutions take the form :
(1) If $c_2 = c_3 = 0$
$u_{40} = a_0 + (\frac{1}{4}\epsilon^2 I c_1 c_4(\sqrt{3} - 3I$
$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1 - I\sqrt{3}}\right)^2))/$$
$$((-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1 - I\sqrt{3}}\right)^2)$$

$v_{40} = -a_0 c - \frac{1}{6}c^2 + (\frac{1}{4}\epsilon^2 I c_1 c c_4(\sqrt{3} - 3I$

$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))/$$

$$((-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2),$$

$w_{40}=h_0.$

(2) If $c_3=c_4=0$

$$u_{41}=a_0-\frac{4\epsilon^2c_1c_2^{\frac{3}{2}}(e^{(\sqrt{c_2}\epsilon\zeta)})}{(4(e^{(\sqrt{c_2}\epsilon\zeta)})^2c_2-4c_1(e^{(\sqrt{c_2}\epsilon\zeta)})\sqrt{c_2}+c_1^2)},$$

$$v_{41}=-\tfrac{1}{3}\epsilon^2c_2c-a_0c-\tfrac{1}{6}c^2-\frac{4\epsilon^2c_1cc_2^{\frac{3}{2}}(e^{(\sqrt{c_2}\epsilon\zeta)})}{(4(e^{(\sqrt{c_2}\epsilon\zeta)})^2c_2-4c_1(e^{(\sqrt{c_2}\epsilon\zeta)})\sqrt{c_2}+c_1^2)},$$

$w_{41}=h_0.$

Case 12. $\{k_2=0,b_2=0,h_0=\dfrac{2g_0+c^2}{4\sqrt{-\frac{1}{2}c}},g_0=g_0,c_4=c_4,c_2=c_2,c_3=c_3,$
$c_1=c_1,a_2=0,g_1=0,g_2=0,d_2=0,a_1=0,h_1=0,h_2=0,$
$a_0=\frac{1}{3}c-\frac{1}{3}\epsilon^2c_2,b_1=-\epsilon^2c_1,d_1=\epsilon^2cc_1,k_1=\sqrt{-\frac{1}{2}c\epsilon^2c_1},c_0=0\}.$

Then φ'will be
$\varphi'=\epsilon\sqrt{c_1\varphi+c_2\varphi^2+c_3\varphi^3+c_4\varphi^4}$
The solitry wave solutions take the form :
(1) If $c_2=c_3=0$
$u_{42}=\frac{1}{3}c+(\frac{1}{2}\epsilon^2Ic_1c_4(\sqrt{3}-3I$

$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))/$$

$$((-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2)$$

$v_{42}=g_0+(\frac{1}{2}\epsilon^2Ic_1cc_4(\sqrt{3}-3I$

$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))/$$

$$((-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2),$$

$$w_{42}=\frac{2g_0+c^2}{4\sqrt{-\frac{1}{2}c}}-(\tfrac{1}{2}\sqrt{-\tfrac{1}{2}c}\epsilon^2Ic_1c_4(\sqrt{3}-3I$$

$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))/$$

$$((-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_3^2)^{\frac{1}{3}}},\frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2).$$

(2) If $c_3 = c_4 = 0$

$$u_{43} = \tfrac{1}{3}c - \tfrac{1}{3}\epsilon^2 c_2 - \frac{8\epsilon^2 c_1 c_2^{\frac{3}{2}}(e^{(\sqrt{c_2}\epsilon\zeta)})}{(4(e^{(\sqrt{c_2}\epsilon\zeta)})^2c_2 - 4c_1(e^{(\sqrt{c_2}\epsilon\zeta)})\sqrt{c_2}+c_1^2)},$$

$$v_{43} = g_0 - \frac{8\epsilon^2 c_1 c c_2^{\frac{3}{2}}(e^{(\sqrt{c_2}\epsilon\zeta)})}{(4(e^{(\sqrt{c_2}\epsilon\zeta)})^2c_2 - 4c_1(e^{(\sqrt{c_2}\epsilon\zeta)})\sqrt{c_2}+c_1^2)},$$

$$w_{43} = \frac{2g_0+c^2}{4\sqrt{-\frac{1}{2}c}} + \frac{8\epsilon^2 c_1\sqrt{-\frac{1}{2}c}c_2^{\frac{3}{2}}(e^{(\sqrt{c_2}\epsilon\zeta)})}{(4(e^{(\sqrt{c_2}\epsilon\zeta)})^2c_2 - 4c_1(e^{(\sqrt{c_2}\epsilon\zeta)})\sqrt{c_2}+c_1^2)}.$$

Case 13. $\{c_0 = c_0, k_2 = 0, a_0 = a_0, c_4 = c_4, h_0 = h_0, c_2 = c_2, c_1 = c_1, a_2 = 0, g_1 = 0, g_2 = 0, a_1 = 0, h_1 = 0, h_2 = 0, k_1 = 0, b_1 = -\epsilon^2 c_1, d_1 = -\epsilon^2 cc_1, b_2 = -2\epsilon^2 c_0, d_2 = -2\epsilon^2 cc_0, g_0 = -\dfrac{c(12a_0c_0 - 3c_1^2\epsilon^2 + 2cc_0 + 16\epsilon^2 c_0 c_2)}{12c_0}, c_3 = -\dfrac{c_1(c_1^2-4c_0c_2)}{8c_0^2}\}.$

Then φ'will be

$$\varphi' = \epsilon\sqrt{c_0 + c_1\varphi + c_2\varphi^2 - \frac{c_1(c_1^2-4c_0c_2)}{8c_0^2}\varphi^3 + c_4\varphi^4}$$

The solitry wave solutions take the form :

(1) If $c_1 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 < 0, c_4 > 0$

$$u_{44} = a_0 + \frac{c_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$

$$v_{44} = -\frac{c(\frac{3a_0c_2^2}{c_4} + \frac{cc_2^2}{2c_4} + \frac{4\epsilon^2c_2^3}{c_4})c_4}{3c_2^2} + \frac{cc_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$

$$w_{44} = h_0.$$

(2) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2m^2(1-m^2)}{c_4(2m^2-1)^2}, c_2 > 0, c_4 < 0$

$$u_{46} = a_0 + \frac{2\epsilon^2 c_2(1-m^2)}{(2m^2-1)cn\left(\sqrt{\frac{c_2}{2m^2-1}}\zeta\right)^2},$$

$$v_{46} = -\frac{1}{12c_2^2m^2(1-m^2)}(c(\frac{12a_0c_2^2m^2(1-m^2)}{c_4(2m^2-1)^2} + \frac{2cc_2^2m^2(1-m^2)}{c_4(2m^2-1)^2}$$

$$+\frac{16\epsilon^2 c_2^3 m^2(1-m^2)}{c_4(2m^2-1)^2})c_4(2m^2-1)^2)+\frac{2\epsilon^2 c_2(1-m^2)}{(2m^2-1)cn\left(\sqrt{\frac{c_2}{2m^2-1}}\zeta\right)^2},$$

$w_{46}=h_0.$

(3) If $c_1=c_3=0, c_0=\frac{c_2^2(1-m^2)}{c_4(2-m^2)^2}, c_2>0, c_4<0$

$$u_{47}=a_0+\frac{2\epsilon^2 c_2(1-m^2)}{(2-m^2)dn\left(\sqrt{\frac{c_2}{2-m^2}}\zeta\right)^2},$$

$$v_{47}=-\frac{1}{12c_2^2(1-m^2)}(c(\frac{12a_0c_2^2(1-m^2)}{c_4(2-m^2)^2}+\frac{2cc_2^2(1-m^2)}{c_4(2-m^2)^2}$$
$$+\frac{16\epsilon^2 c_2^3(1-m^2)}{c_4(2-m^2)^2})c_4(2-m^2)^2)+\frac{2\epsilon^2 cc_2(1-m^2)}{(2-m^2)dn\left(\sqrt{\frac{c_2}{2-m^2}}\zeta\right)^2},$$

$w_{47}=h_0.$

Case 14. $\{c_0=c_0, k_1=k_1, h_0=\frac{2g_0+c^2}{2\sqrt{-2c}}, g_0=g_0, c_4=c_4, c_2=c_2, a_2=0,$
$g_1=0, g_2=0, a_1=0, h_1=0, h_2=0, b_1=-\frac{2k_1}{\sqrt{-2c}}, d_1=k_1\sqrt{-2c},$
$c_1=\frac{k_1}{\sqrt{-2c}\epsilon^2}, k_2=2\sqrt{-2c}\epsilon^2c_0, b2=-4\epsilon^2c_0, d_2=-4\epsilon^2cc_0,$
$a_0=\frac{-3k_1^2+8\epsilon^2c^2c_0-32\epsilon^4cc_0c_2}{24\epsilon^2cc_0}, c_3=\frac{k_1(k_1^2+8\epsilon^4cc_0c_2)}{16c_0^2\sqrt{-2c}\epsilon^6c}\}$

Then φ' will be

$$\varphi'=\epsilon\sqrt{c_0+\frac{k_1}{\sqrt{-2c}\epsilon^2}\varphi+c_2\varphi^2+\frac{k_1(k_1^2+8\epsilon^4cc_0c_2)}{16c_0^2\sqrt{-2c}\epsilon^6c}\varphi^3+c_4\varphi^4}$$

The solitary wave solutions take the form :

(1) If $k_1=0, c_0=\frac{c_2^2}{4c_4}, c_2<0, c_4>0$

$$u_{50}=\frac{c-4\epsilon^2c_2}{3}+\frac{2c_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$
$$v_{50}=g_0+\frac{2cc_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$
$$w_{50}=\frac{2g_0+c^2}{2\sqrt{-2c}}-\frac{\sqrt{-2c}c_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2}.$$

(2) If $k_1=0, c_0=\frac{c_2^2}{4c_4}, c_2>0, c_4>0$

$$u_{51}=\frac{c-4\epsilon^2c_2}{3}-\frac{2c_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2},$$

$$v_{51} = g_0 - \frac{2cc_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2},$$
$$w_{51} = \frac{2g_0 + c^2}{2\sqrt{-2c}} + \frac{\sqrt{-2c}c_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}.$$

(3) If $k_1 = 0, c_0 = \dfrac{c_2^2 m^2(1-m^2)}{c_4(2m^2-1)^2}, c_2 > 0, c_4 < 0$

$$u_{52} = \frac{c - 4\epsilon^2 c_2}{3} + \frac{4\epsilon^2 c_2(1-m^2)}{(2m^2-1)cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2},$$
$$v_{52} = g_0 + \frac{4\epsilon^2 cc_2(1-m^2)}{(2m^2-1)cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2},$$
$$w_{52} = \frac{2g_0 + c^2}{2\sqrt{-2c}} - \frac{2\sqrt{-2c}\epsilon^2 c_2(1-m^2)}{(2m^2-1)cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}.$$

(4) If $c_1 = c_3 = 0, c_0 = \dfrac{c_2^2 m^2}{c_4(1+m^2)^2}, c_2 < 0, c_4 > 0$

$$u_{54} = \frac{c - 4\epsilon^2 c_2}{3} + \frac{4\epsilon^2 c_2}{(m^2+1)sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2},$$
$$v_{54} = g_0 + \frac{4\epsilon^2 cc_2}{(m^2+1)sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2},$$
$$w_{54} = \frac{2g_0 + c^2}{2\sqrt{-2c}} - \frac{2\sqrt{-2c}\epsilon^2 c_2}{(m^2+1)sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2}.$$

Case 15. $\{k_2 = 0, b_2 = 0, a_0 = a_0, c_4 = c_4, h_0 = h_0, c_2 = c_2, c_3 = c_3, b_1 = 0, d_1 = 0, d_2 = 0, h_1 = 0, h_2 = 0, k_1 = 0, g_1 = -\epsilon^2 cc_3, a_1 = -\epsilon^2 c_3, a_2 = -2\epsilon^2 c_4, g_2 = -2\epsilon^2 cc_4, g_0 = -\dfrac{c(2cc_4 + 16\epsilon^2 c_2 c_4 - 3\epsilon^2 c_3^2 + 12a_0 c_4)}{12c_4}, c_1 = \dfrac{c_3(4c_2c_4 - c_3^2)}{8c_4^2}, c_0 = 0\}$

Then φ'will be

$\varphi' = \epsilon\sqrt{\frac{c_3(4c_2c_4-c_3^2)}{8c_4^2}\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}$

The solitary wave solution take the form :

(1) If $c_2 = 0$

$$u_{55} = a_0 + \frac{\epsilon^2 c_3^2 sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5} + \frac{3}{2})^2}{c_4\left(\sqrt{5} - \frac{3}{2} + 2sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5} + \frac{3}{2})^2\right)}$$
$$- \frac{2\epsilon^2 c_3^2 sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5} + \frac{3}{2})^4}{c_4\left(\sqrt{5} - \frac{3}{2} + 2sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5} + \frac{3}{2})^2\right)^2},$$

$$v_{55} = \frac{c(2cc_4 - 3\epsilon^2 c_3^2 + 12a_0c_4)}{12c_4} + \frac{\epsilon^2 cc_3^2 sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5}+\frac{3}{2})^2}{c_4\left(\sqrt{5}-\frac{3}{2}+2sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5}+\frac{3}{2})^2\right)}$$
$$-\frac{2\epsilon^2 c_3^2 sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5}+\frac{3}{2})^4}{c_4\left(\sqrt{5}-\frac{3}{2}+2sn(\frac{\sqrt{2}c_3\sqrt{-c_4(-5+3)}\epsilon\zeta}{8c_4}, \frac{1}{2}\sqrt{5}+\frac{3}{2})^2\right)^2},$$
$w_{55} = h_0.$

Case 16. $\{k_2 = 0, b_2 = 0, a_0 = a_0, c_4 = c_4, h_0 = h_0, c_3 = c_3, c_1 = c_1, d2 = 0,$ $h_1 = 0, h_2 = 0, k_1 = 0, g_1 = -\epsilon^2 cc_3, a_1 = -\epsilon^2 c_3, a_2 = -2\epsilon^2 c_4, g_2 = -2\epsilon^2 cc_4,$ $c_0 = 0, g_0 = -\dfrac{c(\epsilon^2 c_3^2 + +2cc_4 + 12a_0c_4)}{12c_4}, c_2 = \frac{c_3^2}{4c_4}, b_1 = -\frac{1}{2}\epsilon^2 c_1, d_1 = -\frac{1}{2}\epsilon^2 cc_1\}.$

Then φ'will be

$\varphi' = \epsilon\sqrt{c_1\varphi + \frac{c_3^2}{4c_4}\varphi^2 + c_3\varphi^3 + c_4\varphi^4}$

The solitary wave solution take the form :

(1) If $c_3 = 0$

$$u_{57} = a_0 + (8\epsilon^2(-c_1c_4^2)^{\frac{2}{3}} sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^4)/$$
$$\left(c_4\left(\sqrt{3}-3I+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2\right)\right)^2$$
$$+(\tfrac{1}{4}I\epsilon^2 c_1c_4(\sqrt{3}-3+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))^2/$$
$$(-c_1c_4^2)^{\frac{1}{3}} sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right),$$
$$v_{57} = -\frac{2c^2\epsilon^2c_4 - 2h_1^2}{4\epsilon^2c_4} + (8\epsilon^2 c(-c_1c_4^2)^{\frac{2}{3}}$$
$$sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^4)/$$
$$\left(c_4\left(\sqrt{3}-3I+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2\right)\right)^2$$
$$+(\tfrac{1}{4}I\epsilon^2 cc_1c_4(\sqrt{3}-3I+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))^2/$$
$$(-c_1c_4^2)^{\frac{1}{3}} sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right),$$
$w_{57} = h_0.$

Case 17. $\{c_0 = c_0, k_2 = 0, b_2 = 0, a_0 = a_0, c_4 = c_4, h_0 = h_0, c_2 = c_2, c_3 = c_3,$ $b_1 = 0, d_1 = 0, d_2 = 0, h_1 = 0, h_2 = 0, k_1 = 0, a_1 = -\epsilon^2 c_3, a_2 = -2\epsilon^2 c_4,$ $g_2 = -2\epsilon^2 cc_4, g_0 = -\dfrac{c(16\epsilon^2 c_2 c_4 + 2cc_4 - 3\epsilon^2 c_3^2 + 12a_0 c_4)}{12c_4}, g_1 = -\epsilon^2 cc_3,$ $c_1 = \dfrac{c_3(4c_2c_4 - c_3^2)}{8c_4^2}\}.$

Then φ' will be

$$\varphi' = \epsilon\sqrt{c_0 + \frac{c_3(4c_2c_4 - c_3^2)}{8c_4^2}\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}$$

The solitary wave solutions take the form :

(1) If $c_3 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 < 0, c_4 > 0$

$$u_{58} = a_0 + \epsilon^4 c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2,$$

$$v_{58} = -\frac{c(16\epsilon^2 c_2 c_4 + 2cc_4 + 12a_0c_4)}{12c_4} + \epsilon^4 cc_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2,$$

$$w_{58} = h_0.$$

(2) If $c_3 = 0, c_0 = \dfrac{c_2^2}{4c_4}, c_2 > 0, c_4 > 0$

$$u_{59} = a_0 - \epsilon^4 c_2 \tan(\tfrac{1}{2}\sqrt{2c_2}\zeta)^2,$$

$$v_{59} = -\frac{c(16\epsilon^2 c_2 c_4 + 2cc_4 + 12a_0c_4)}{12c_4} - \epsilon^4 cc_2 \tan(\tfrac{1}{2}\sqrt{2c_2}\zeta)^2,$$

$$w_{59} = h_0.$$

(3) If $c_3 = 0, c_0 = \dfrac{c_2^2 m^2(1-m^2)}{c_4(2m^2-1)^2}, c_2 > 0, c_4 < 0$

$$u_{60} = a_0 + \frac{2\epsilon^2 c_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1},$$

$$v_{60} = -\frac{c(16\epsilon^2 c_2 c_4 + 2cc_4 + 12a_0c_4)}{12c_4} + \frac{2\epsilon^2 cc_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1},$$

$$w_{60} = h_0.$$

(4) If $c_3 = 0, c_0 = \dfrac{c_2^2 m^2}{c_4(1+m^2)^2}, c_2 < 0, c_4 > 0$

$$u_{62} = a_0 + \frac{2\epsilon^2 c_2 m^2 sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2}{m^2+1},$$

$$v_{62} = -\frac{c(16\epsilon^2 c_2 c_4 + 2cc_4 + 12a_0c_4)}{12c_4} + \frac{2\epsilon^2 cc_2 m^2 sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2}{m^2+1},$$

$$w_{62} = h_0.$$

Case 18. $\{c_0 = c_0, k_2 = 0, a_0 = a_0, c_4 = c_4, h_0 = h_0, c_2 = c_2, b_1 = 0, g_1 = 0,$ $d_1 = 0, a_1 = 0, h_1 = 0, h_2 = 0, k_1 = 0, g_0 = -\frac{1}{6}c(6a_0 + c + 8\epsilon^2 c_2), c_3 = 0,$ $a_2 = -2\epsilon^2 c_4, g_2 = -2\epsilon^2 cc_4, c_1 = 0, b_2 = -2\epsilon^2 c_0, d_2 = -2\epsilon^2 cc_0\}$

Then φ'will be
$\varphi' = \epsilon\sqrt{c_0 + c_2\varphi^2 + c_4\varphi^4}$
The solitary wave solutions take the form :
(1) If $c_0 = \frac{c_2^2}{4c_4}, c_2 < 0, c_4 > 0$
$u_{64} = a_0 + \epsilon^4 c_2 \tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2 + \frac{c_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2}$,
$v_{64} = -\frac{1}{6}c(6a_0 + c) + \epsilon^4 cc_2 \tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2 + \frac{cc_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2}$,
$w_{64} = h_0$.
(2) If $c_0 = \frac{c_2^2}{4c_4}, c_2 > 0, c_4 > 0$
$u_{65} = a_0 - \epsilon^4 c_2 \tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2 - \frac{c_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}$,
$v_{65} = -\frac{1}{6}c(6a_0 + c) - \epsilon^4 cc_2 \tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2 - \frac{cc_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}$,
$w_{65} = h_0$.
(3) If $c_0 = \frac{c_2^2(1-m^2)}{c_4(2-m^2)^2}, c_2 > 0, c_4 < 0$

$$u_{67} = a_0 + \frac{2\epsilon^2 c_2 dn(\sqrt{\frac{c_2}{2-m^2}}\zeta)^2}{2-m^2} + \frac{2\epsilon^2 c_2(1-m^2)}{(2-m^2)dn(\sqrt{\frac{c_2}{2-m^2}}\zeta)^2},$$

$$v_{67} = -\frac{1}{6}c(6a_0 + c + 8\epsilon^2 c_2) + \frac{2\epsilon^2 cc_2 dn(\sqrt{\frac{c_2}{2-m^2}}\zeta)^2}{2-m^2} + \frac{2\epsilon^2 cc_2(1-m^2)}{(2-m^2)dn(\sqrt{\frac{c_2}{2-m^2}}\zeta)^2},$$

$w_{67} = h_0$.

Case 19. $\{k_2 = 0, b_2 = 0, c_4 = c_4, c_2 = c_2, c_1 = c_1, g_1 = 0, d_2 = 0, a_1 = 0,$
$h_1 = 0, k_1 = 0, g_0 = \epsilon^2 c_2 c - \frac{1}{2}c^2, a_0 = \frac{1}{3}c - \frac{4}{3}\epsilon^2 c_2, h_0 = \frac{\epsilon^2 c_2 c}{\sqrt{-2c}}, c_3 = 0,$
$h_2 = 2\sqrt{-2c}\epsilon^2 c_4, a_2 = -4\epsilon^2 c_4, g_2 = -4\epsilon^2 cc_4, b_1 = -\frac{1}{2}\epsilon^2 c_1,$
$d_1 = -\frac{1}{2}\epsilon^2 cc_1, c_0 = 0\}$.

Then φ'will be
$\varphi' = \epsilon\sqrt{c_1\varphi + c_2\varphi^2 + c_4\varphi^4}$
The solitary wave solution take the form :

(1) If $c_2 = 0$

$$u_{69} = \frac{1}{3}c + (16\epsilon^2(-c_1c_4^2)^{\frac{2}{3}} sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^4)/$$
$$\left(c_4\left(\sqrt{3} - 3I + 2I sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)\epsilon\zeta}}{4(-c_0c_4^2)^{\frac{1}{3}}}, \frac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2\right)\right)^2$$

$$+(\tfrac{1}{4}I\epsilon^2 cc_1c_4(\sqrt{3}-3I$$
$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))^2/$$
$$(-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2,$$
$$v_{69}=-\frac{2c^2\epsilon^2c_4-2h_1^2}{4\epsilon^2c_4}$$
$$+(16\epsilon^2c(-c_1c_4^2)^{\frac{2}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^4)/$$
$$\left(c_4\left(\sqrt{3}-3I+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2\right)\right)^2$$
$$+(\tfrac{1}{4}I\epsilon^2 cc_1c_4(\sqrt{3}-3I$$
$$+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2))^2/$$
$$(-c_1c_4^2)^{\frac{1}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2,$$
$$w_{69}=-(8\epsilon^2\sqrt{-2c}(-c_1c_4^2)^{\frac{2}{3}}sn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^4)$$
$$/\left(c_4\left(\sqrt{3}-3I+2Isn\left(\frac{\frac{1}{4}I\sqrt{2}\sqrt{-I(-c_1c_4^2)^{\frac{1}{3}}c_1c_4(-\sqrt{3}+3I)}\epsilon\zeta}{4(-c_0c_4^2)^{\frac{1}{3}}},\tfrac{1}{2}\sqrt{2}\sqrt{1-I\sqrt{3}}\right)^2\right)\right)^2.$$

Case 20. $\{c_0=c_0, k_2=0, b_2=0, h_1=h_1, g_0=g_0, c_4=c_4, c_2=c_2, b_1=0,$
$d_1=0, d_2=0, h_0=\dfrac{2g_0+c^2}{2\sqrt{-2c}}, a_1=\dfrac{-2h_1}{\sqrt{-2c}}, g_1=h_1\sqrt{-2c}, k_1=0,$
$c_3=\dfrac{h_1}{\sqrt{-2c}\epsilon^2}, h_2=2\sqrt{-2c}\epsilon^2c_4, a_2=-4\epsilon^2c_4, g_2=-4\epsilon^2cc_4,$
$a_0=\dfrac{-3h_1^2-32\epsilon^4c_2cc_4+8c^2\epsilon^2c_4}{24\epsilon^2c_4c}, c_1=\dfrac{(h_1^2+8\epsilon^4c^2cc_4)h_1}{16c_4^2\sqrt{-2c}\epsilon^6c}\}.$

Then φ'will be
$$\varphi'=\epsilon\sqrt{c_0+\frac{(h_1^2+8\epsilon^4c^2cc_4)h_1}{16c_4^2\sqrt{-2c}\epsilon^6c}\varphi+c_2\varphi^2+\frac{h_1}{\sqrt{-2c}\epsilon^2}\varphi^3+c_4\varphi^4}$$
The solitary wave solutions take the form :
(1) If $h_1=0, c_0=\dfrac{c_2^2}{4c_4}, c_2<0, c_4>0$

$$u_{70} = \frac{-32\epsilon^4 c_2 c c_4 + 8c^2\epsilon^2 c_4}{24\epsilon^2 c_4 c} + 2\epsilon^4 c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2,$$
$$v_{70} = g_0 + 2\epsilon^4 c c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2,$$
$$w_{70} = \frac{2g_0 + c^2}{2\sqrt{-2c}} - \sqrt{-2c}\epsilon^4 c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2.$$

(2) If $h_1 = 0, c_0 = \dfrac{c_2^2 m^2(1-m^2)}{c_4(2m^2-1)^2}, c_2 > 0, c_4 < 0$

$$u_{72} = \frac{-32\epsilon^4 c_2 c c_4 + 8c^2\epsilon^2 c_4}{24\epsilon^2 c_4 c} + \frac{4\epsilon^2 c_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1},$$
$$v_{72} = g_0 + \frac{4\epsilon^2 c c_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1},$$
$$w_{72} = \frac{2g_0 + c^2}{2\sqrt{-2c}} - \frac{2\sqrt{-2c}\epsilon^2 c_2 m^2 cn(\sqrt{\frac{c_2}{2m^2-1}}\zeta)^2}{2m^2-1}.$$

(3) If $c_0 = \dfrac{c_2^2 m^2}{c_4(1+m^2)^2}, c_2 < 0, c_4 > 0$

$$u_{74} = \frac{-32\epsilon^4 c_2 c c_4 + 8c^2\epsilon^2 c_4}{24\epsilon^2 c_4 c} + \frac{4\epsilon^2 c_2 m^2 sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2}{m^2+1},$$
$$v_{74} = g_0 + \frac{4\epsilon^2 c c_2 m^2 sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2}{m^2+1},$$
$$w_{74} = \frac{2g_0 + c^2}{2\sqrt{-2c}} - \frac{2\sqrt{-2c}\epsilon^2 c_2 m^2 sn(\sqrt{-\frac{c_2}{m^2+1}}\zeta)^2}{m^2+1}.$$

Case 21. $\{c_0 = c_0, a_2 = \dfrac{h_2\sqrt{-2c}}{c}, h_2 = h_2, h_0 = \dfrac{2g_0 + c^2}{2\sqrt{-2c}}, g_0 = g_0, c_2 = c_2,$
$b_1 = 0, g_1 = 0, d_1 = 0, a_1 = 0, h_1 = 0, g_2 = h_2\sqrt{-2c}, k_1 = 0,$
$a_0 = \frac{1}{3}c - \frac{4}{3}\epsilon^2 c_2, c_3 = 0, c_4 = \dfrac{h_2}{2\sqrt{-2c}\epsilon^2}, c_1 = 0, k_2 = 2\sqrt{-2c}\epsilon^2 c_0,$
$b_2 = -4\epsilon^2 c_0, d_2 = -4\epsilon^2 c c_0\}.$

Then φ' will be

$\varphi' = \epsilon\sqrt{c_0 + c_2\varphi^2 + \frac{h_2}{2\sqrt{-2c}\epsilon^2}\varphi^4}$

The solitary wave solutions take the form :

(1) If $c_0 = \dfrac{c_2^2}{4c_4}, c_4 = \frac{h_2}{2\sqrt{-2c}\epsilon^2}, c_2 < 0, c_4 > 0$

$$u_{75} = -\tfrac{4}{3}\epsilon^2 c_2 + \tfrac{1}{3}c + 2\epsilon^4 c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2 + \frac{2c_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$
$$v_{75} = g_0 + 2\epsilon^4 c c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2 + \frac{2cc_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2},$$

$$w_{75} = \frac{2g_0 + c^2}{2\sqrt{-2c}} - \sqrt{-2c}\epsilon^4 c_2 \tanh(\tfrac{1}{2}\sqrt{-2c_2}\zeta)^2 - \frac{\sqrt{-2}cc_2}{\tanh(\frac{1}{2}\sqrt{-2c_2}\zeta)^2}.$$

(2) If $c_0 = \dfrac{c_2^2}{4c_4}, c_4 = \frac{h_2}{2\sqrt{-2c\epsilon^2}}, c_2 > 0, c_4 > 0$

$$u_{76} = -\tfrac{4}{3}\epsilon^2 c_2 + \tfrac{1}{3}c - 2\epsilon^4 c_2 \tan(\tfrac{1}{2}\sqrt{2c_2}\zeta)^2 - \frac{2c_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2},$$

$$v_{76} = g_0 - 2\epsilon^4 cc_2 \tan(\tfrac{1}{2}\sqrt{2c_2}\zeta)^2 - \frac{2cc_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}, Paper$$

$$w_{76} = \frac{2g_0 + c^2}{2\sqrt{-2c}} + \sqrt{-2c}\epsilon^4 c_2 \tan(\tfrac{1}{2}\sqrt{2c_2}\zeta)^2 + \frac{\sqrt{-2}cc_2}{\tan(\frac{1}{2}\sqrt{2c_2}\zeta)^2}.$$

3. Conclusion

In this paper, the periodic wave solution by the Jacobi elliptic, Weierstrass, elliptic, triangle, and hyperbolic functions and other type of functions for nonlinear generalized Ito system are derived. Therefore, many solitary wave solutions can be obtained. From the derivation of hte periodic wave solutions, it is easy to see that our method is useful tool to explore the periodic solution for our system.

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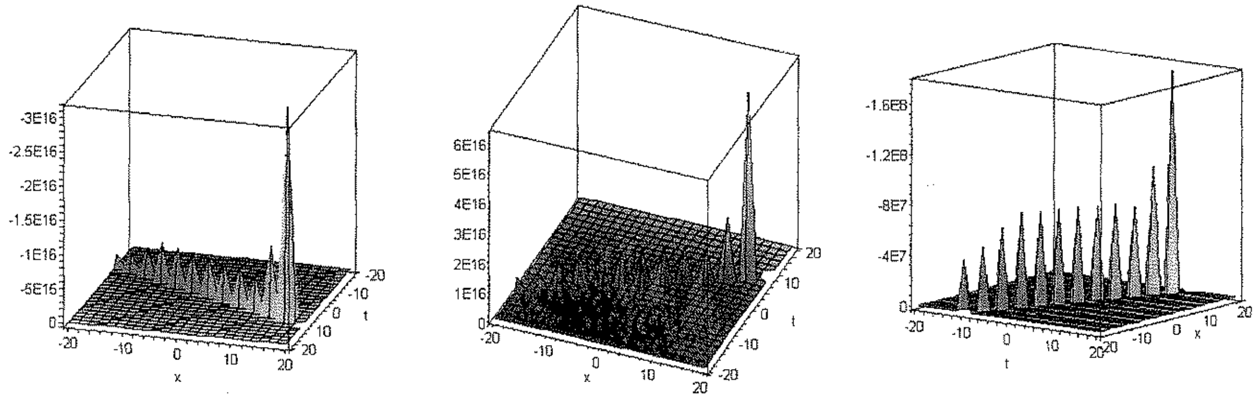
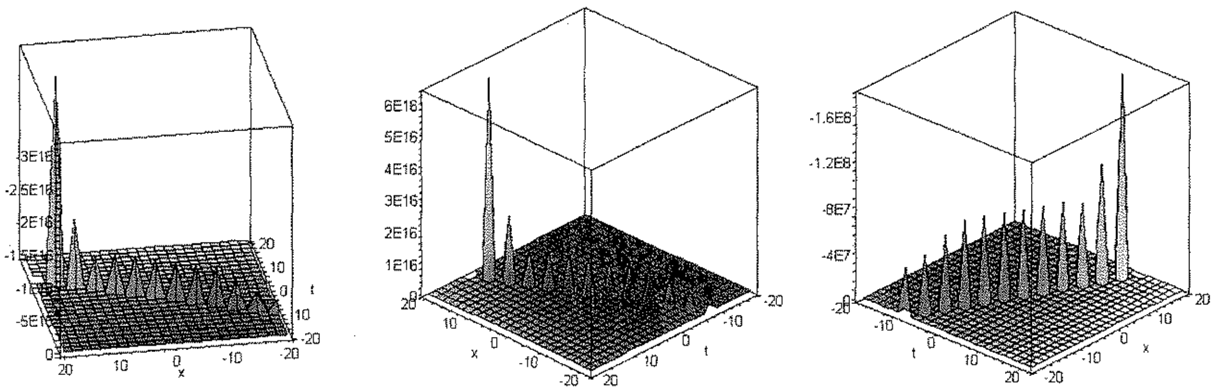
Curves of solutions of Ito coupled system

Case1

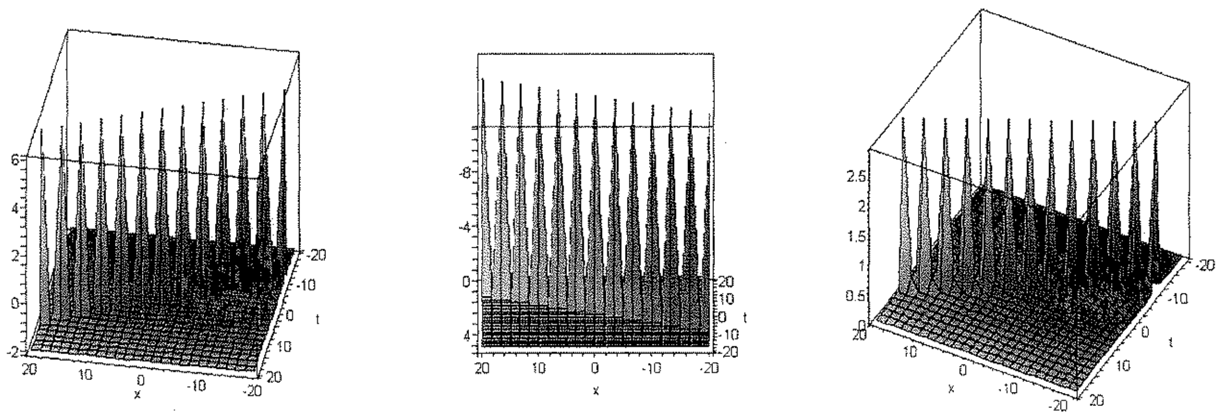


$(u_1\ ,\ v_1\ ,\ w_1)$ **when** $\{c_2=-4, \varepsilon=1, c=-2, c_4=2, k_1=2\}$

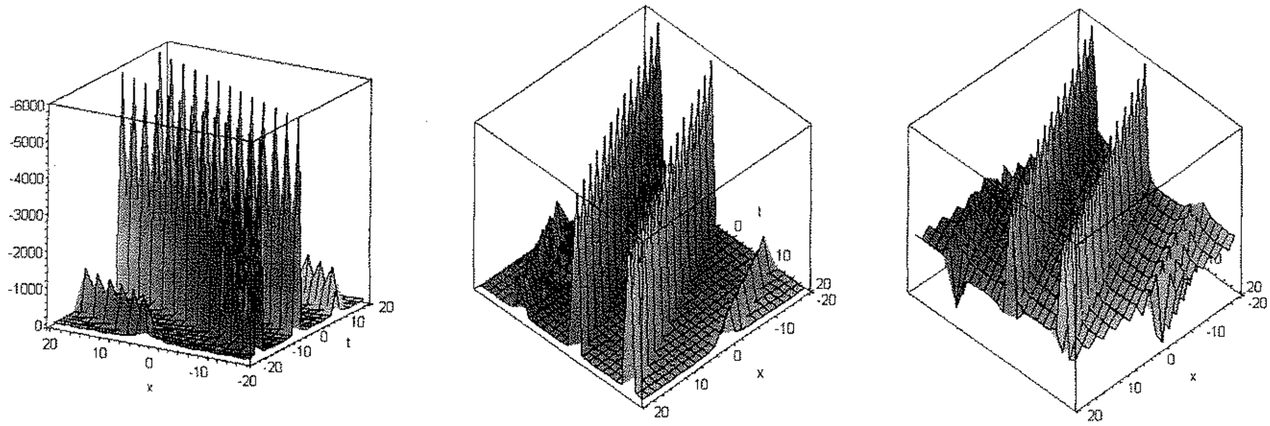


$(u_2\ ,\ v_2\ ,\ w_2)$ **when** $\{c_2=4, g_0=1,\ \varepsilon=1, c=-2, c_4=2, h_0=1, k_1=2\}$

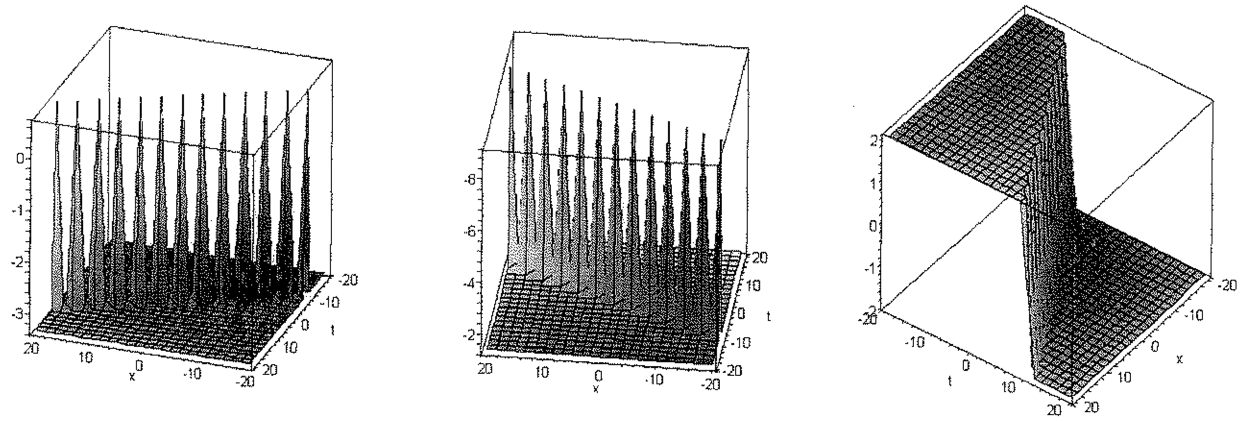
Case2



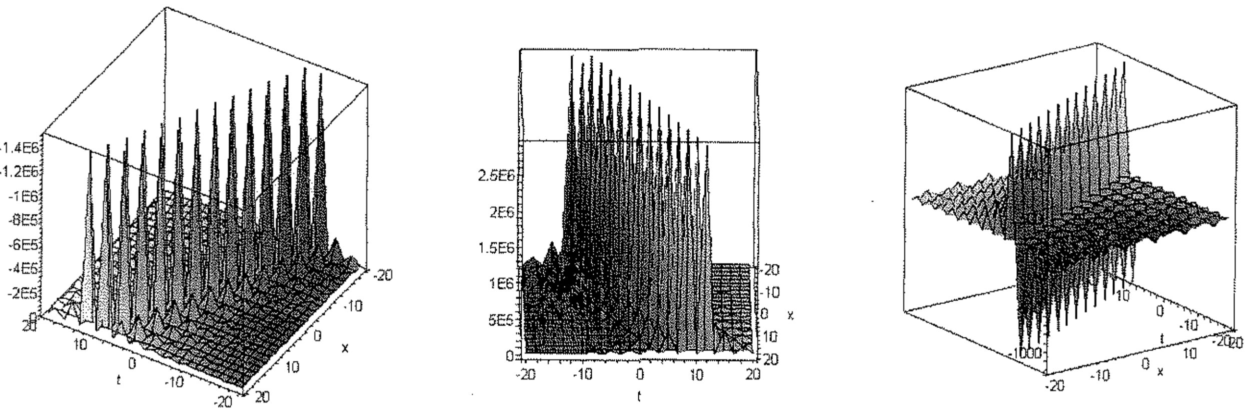
$(u_6\ ,\ v_6\ ,\ w_6)$ **when** $\{c_4=-2, c_2=4,\ \varepsilon=1, c=-2, h_1=2\}$



(u_7 , v_7 , w_7){c_4=2,c_2=-4, ε =1,c=-2,h_1=2}

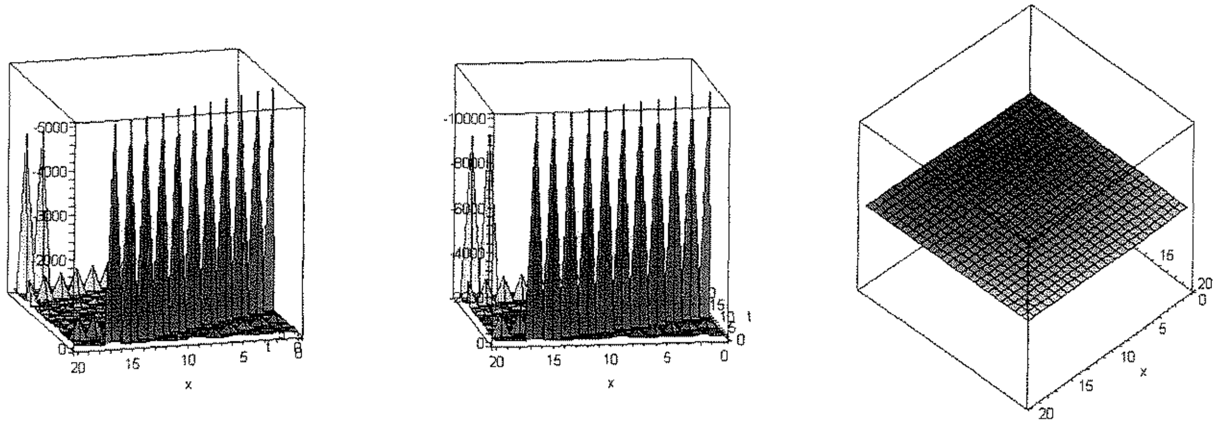


(u_9 , v_9 , w_9) when {c_4=2,c_2=-4, ε =1,c=-2,h_1=2}



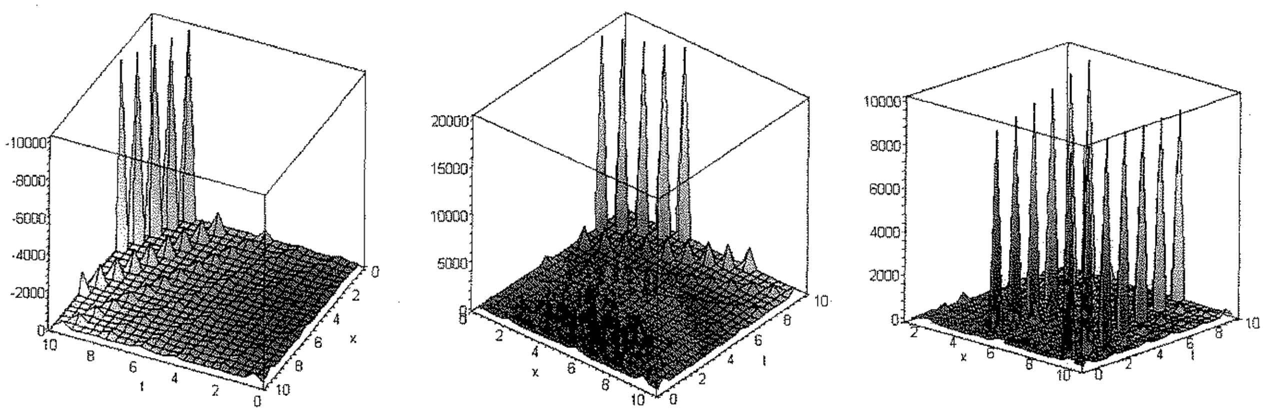
(u_{10} , v_{10} , w_{10}) when {c_4=2,c_2=4, ε =1,c=-2,h(1)=2}

Case3



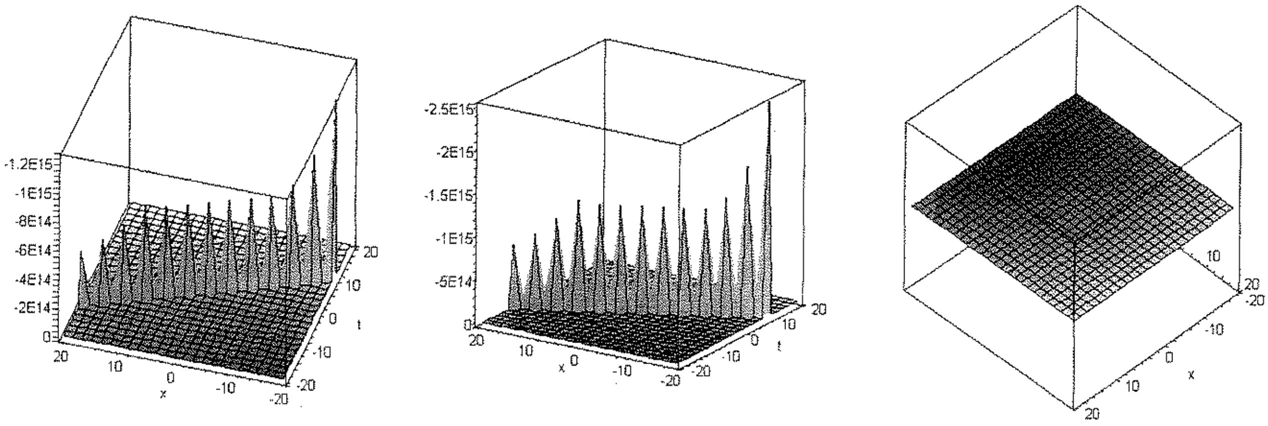
(u_{14}, v_{14}, w_{14}) when $\{a_0=1, c_3=1, c_1=-4, \varepsilon=1, c=2, h_0=1\}$

Case4



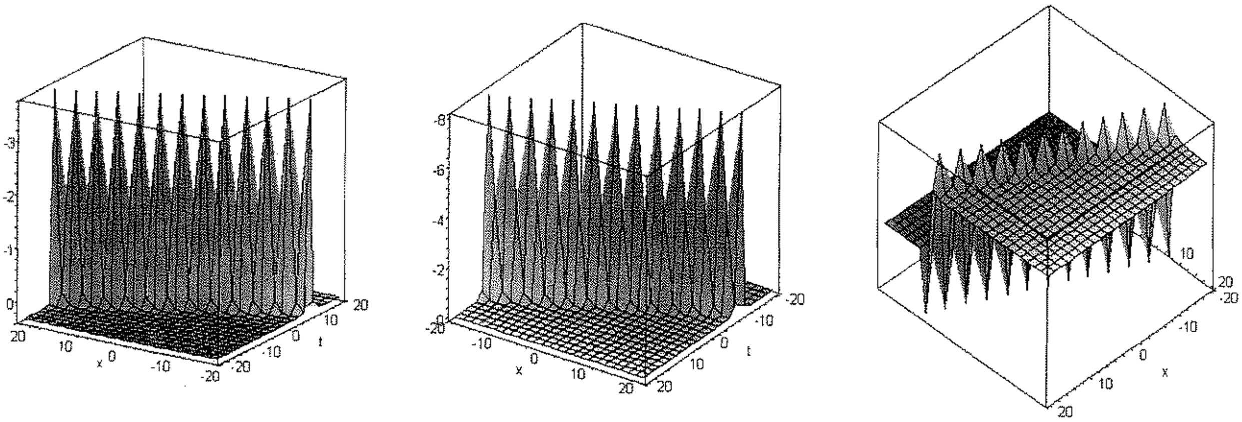
(u_{15}, v_{15}, w_{15}) when $\{a_0=1, c_3=1, c_1=-4, \varepsilon=1, c=-2, h_0=1\}$

Case5



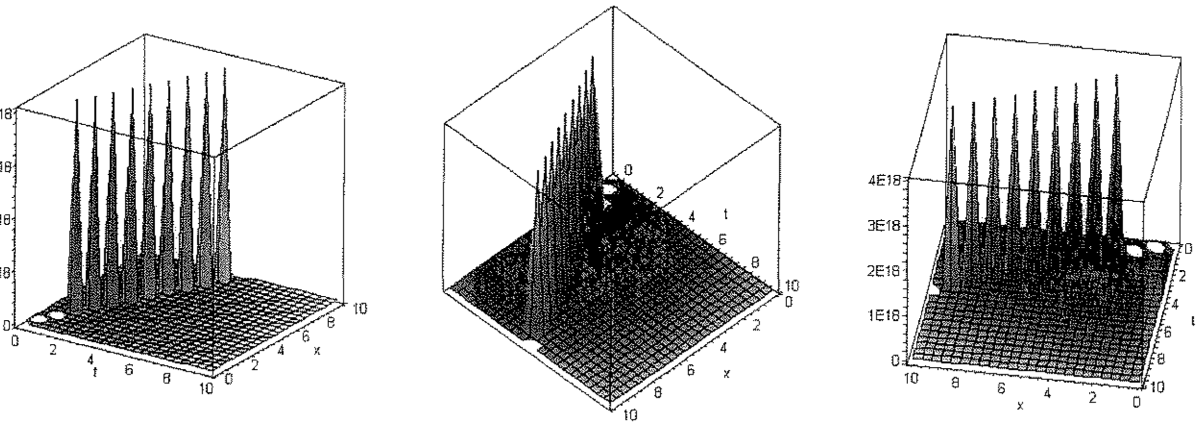
(u_{16}, v_{16}, w_{16}) when $\{c_0=2, c_3=1, c_2=1, \varepsilon=1, c=2, h_0=1, a_0=2\}$

Case6

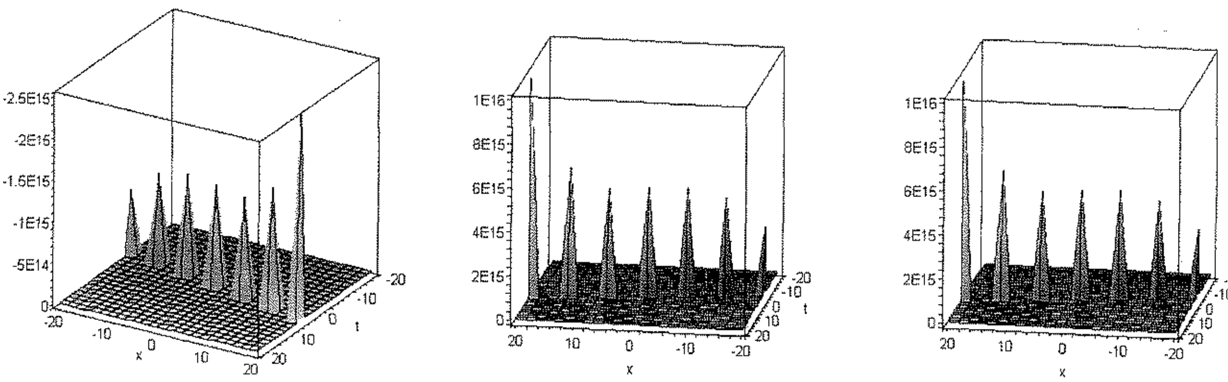


$(u_{18}\ ,\ v_{18}\ ,\ w_{18})$ **when** $\{c_2=1, c_1=2,\ \varepsilon=1, c=2, k_1=2\}$

Case7

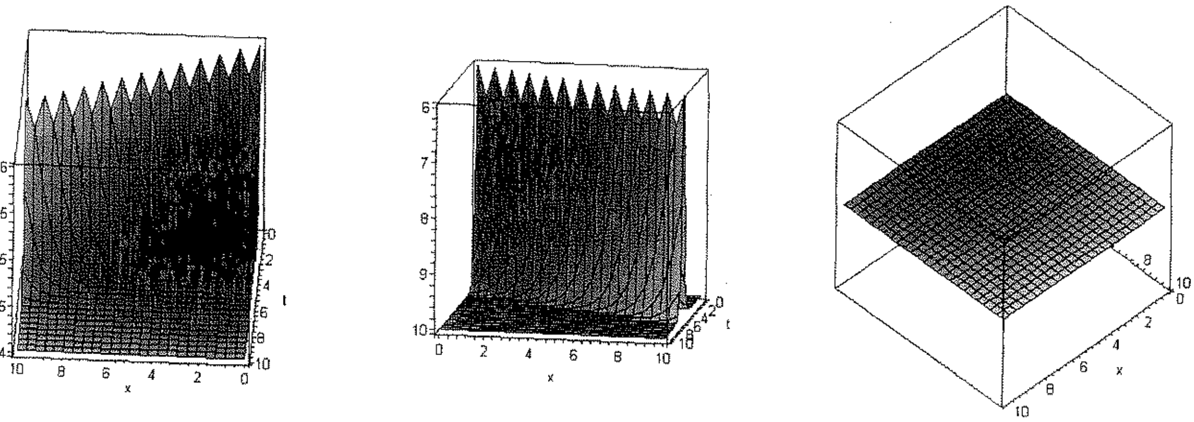


$(u_{20}\ ,\ v_{20}\ ,\ w_{20})$ **when** $\{c_0=4, c_2=4, g_0=1,\ \varepsilon=1, c=-2\}$

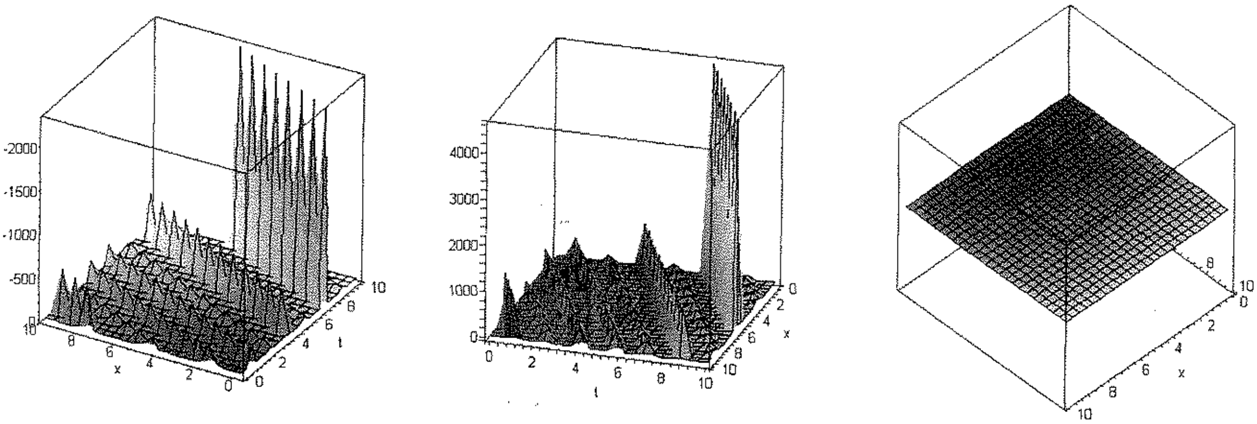


$(u_{22}\ ,\ v_{22}\ ,\ w_{22})$ **when** $\{c_0=4, c_2=4, g_0=1,\ \varepsilon=1, c=-4\}$

Case8

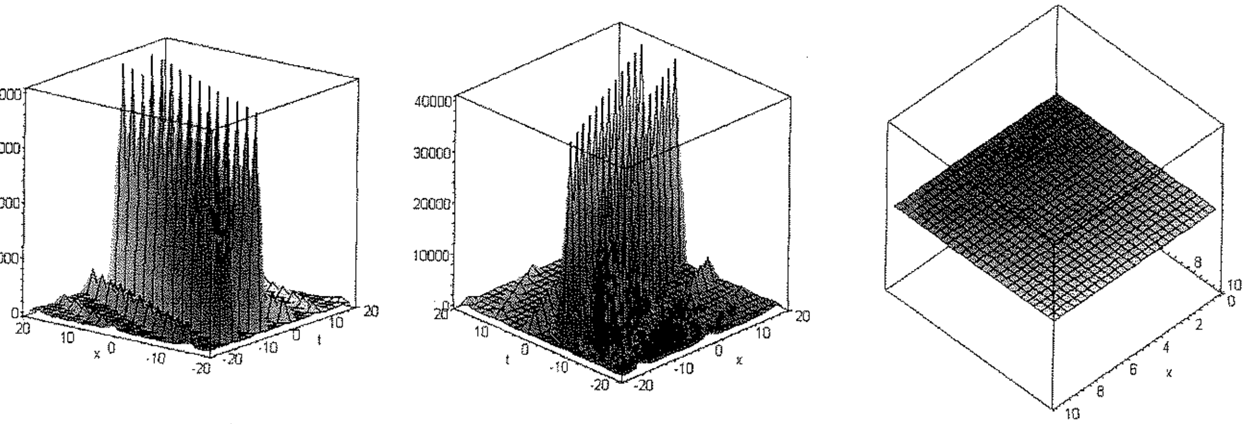


(u_{23}, v_{23}, w_{23}) when $\{a_0=4, c_2=4, \varepsilon=1, c=-2, h_0=1\}$



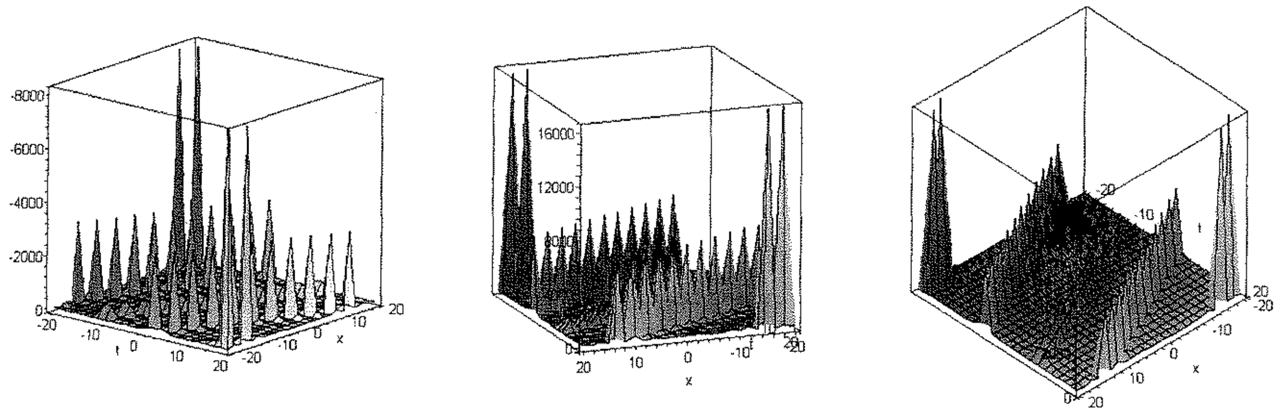
(u_{24}, v_{24}, w_{24}) when $\{a_0=4, c_2=-4, \varepsilon=1, c=-2, h_0=1\}$

Case 9



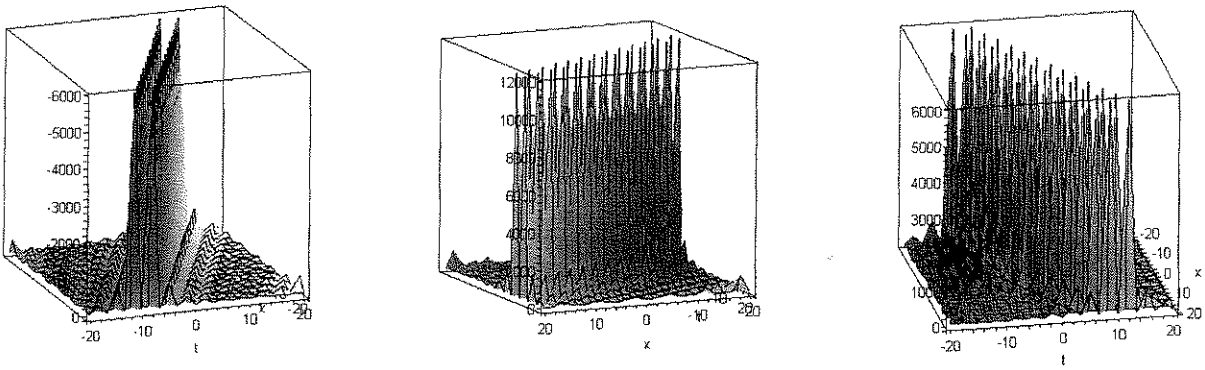
(u_{27}, v_{27}, w_{27}) when $\{a_0=4, c_1=-4, \varepsilon=1, c=-2, h_0=1, c_3=16\}$

Case13



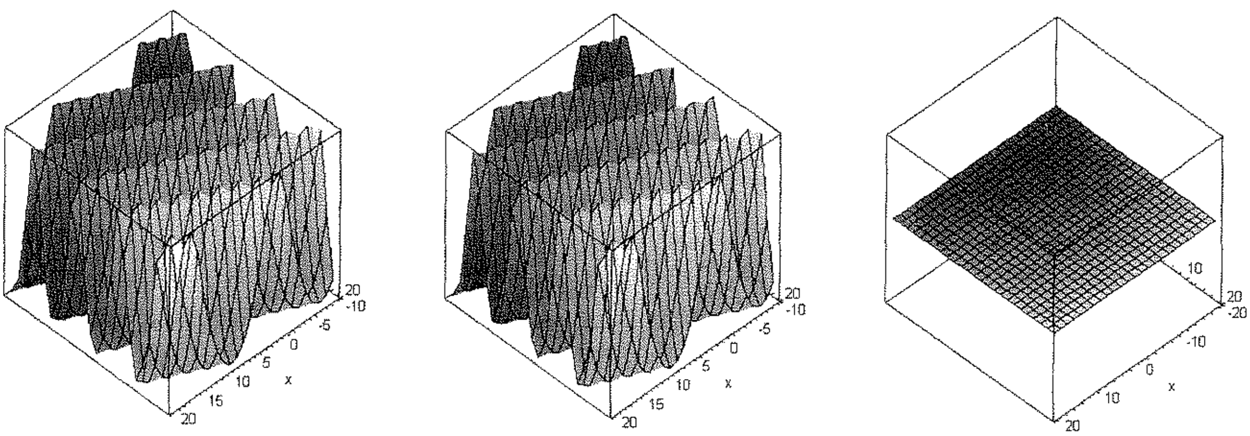
(u_{34} , v_{34} , w_{34}) when {c_2=-4,g_0=1, ε =1,c=-2}

Case 14



(u_{37} , v_{37} , w_{37}) when {h_1=8,g_0=1,c_1=-4, ε =1,c=-2}

Case 21



(u_{55} , v_{55} , w_{55}) when {c_3=1, ε =1,c=2,h_0=1,a_0=2,c_4=-4}