Linear Time Recognition of Bipartite $Star_{123}$-Free Graphs

Ruzayn Quaddoura
Faculty of Science and Information Technology, Zarqa Private University, Jordan

Abstract: *In this paper, we present a linear time recognition algorithm for recognizing bipartite graphs without induced subgraphs isomorph to $star_{123}$. Bipartite $star_{123}$-free graphs are a natural generalization of both weak bisplit and $Star_{123}$, Sun_4-free bipartite graphs, both further generalizing bicographs.*

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1. Introduction

Bipartite graphs underlie suitable models for such a broad spectrum of real-life problems, that adapting results on general graphs to the bipartite case and even attacking combinatorial problems separately for bipartite graphs has been a sufficiently motivated activity since the early years of graph theory; see for example [12] where a bipartite translation" for various graph concepts is given.

This is also the case for the definition and characterization of many special graph classes; for instance, Frost *et al.* in [5] propose several bipartite analogues of split graphs. Giakoumakis and Vanherpe defined in [6] the class of bicographs as a bipartite equivalent of cographs and showed that the bicographs are exactly the class of $Star_{123}$, Sun_4, P_7-free graphs.

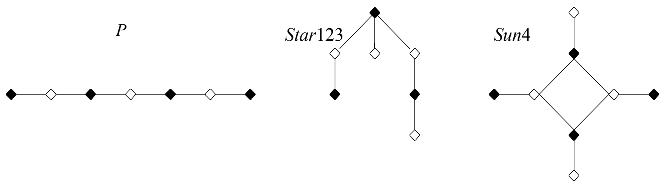

Figure 1. The forbidden configurations for the bicographs.

Fouquet *et al.* in [4] defined the weak bisplit graphs that turned out to be a generalization of bicographs. More precisely, in [4] a general decomposition scheme for bipartite graphs (called canonical decomposition) is given, under which the weak-bisplit graphs are totally decomposable; it is finally shown that these graphs are exactly the bipartite graphs with no induced $Star_{123}$ nor P_7. Another generalization of bicographs, the $Star_{123}$, Sun_4-free bipartite graphs, have been studied in [10], but no recognition algorithm is given.

In this paper, we present a linear time algorithm for a further generalization of both weak-bisplit and $Star_{123}$, Sun_4-free bipartites, namely, the $Star_{123}$-free bipartite graphs. To this end, our algorithm extends the recognition algorithm for $Star_{123}$, P_7-free bipartite graphs given in [7], by making use of some structural properties of the $Star_{123}$-free bipartite graphs, first discussed by Lozin in [9]. For simplicity and abbreviation, we will present all the theorems without proofs, the reader can find these proofs in [13].

The paper is organized as follows. In section 2, we are giving the basic concepts and notations to be used throughout this paper. In section 3, we define the extended canonical decomposition for bipartite graphs and we present Lozin's theorem on the structure of the $Star_{123}$-free bipartite graphs. In section 4, the main ideas of the recognition algorithm are presented in the form of procedures and necessary conditions concerning the decomposition tree. In section 5, the final algorithm is given and it is shown that, using suitable data structures, its execution time is linear on the input size. Section 6 concludes the paper.

2. Notation and Terminology

For terms not defined in this paper the reader can refer to [1]. The graphs considered in this paper are finite, without multiple edges or loops. As usual, for any graph G, we denote by $V(G)$ the set of its vertices and by $E(G)$ the set of its edges (or simply by V and E if there is no risk of confusion) by n and m their respective cardinalities. A bipartite graph $G = (B \cup W, E)$ is defined by two disjoint vertex subsets B - the *black* vertices and W - the *white* ones, and a set of edges $E \subseteq B \times W$.

If the color classes B or W are both non empty the graph will be called *bichromatic*, *monochromatic* otherwise. The *bicomplement* of a bipartite graph $G = (B \cup W, E)$ is the bipartite graph defined by $\overline{G}^{bip} = (B \cup W, B \times W - E)$. For a vertex x, the set of its neighbors in G is denoted by $N(x)$, the cardinality of $N(x)$ is called the degree of x in G and is denoted by $d_G(x)$ (or